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JEL H32, P16.

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Anti-agglomeration Subsidies with Heterogeneous Firms

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ABSTRACT

This paper studies anti-agglomeration subsidies in a core-periphery setting when firms are heterogeneous in terms of efficiency, focusing on the positive and normative effects of various subsidy forms (specific versus proportional), various tax-financing schemes (local versus global) and various capital-labour endowment ratios (symmetric versus asymmetric). Anti-agglomeration subsidies are shown to have ambiguous welfare effects and the determinants of the sign and size of the welfare impact are characterised.

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1. INTRODUCTION

One of the central elements of EU policy concerns anti-agglomeration subsidies called 'structural spending'. More specifically, the Structural and Cohesion Funds aim to diminish the wealth gap between the so-called core and periphery, fight against unemployment, and stimulate economic growth in poor regions. About 30% to 40 % of EU expenditure is spent on these anti-agglomeration policies (39% in 2006). In more detail, over 70% of the structural spending is spent on basic infrastructure and production subsidies in poor regions (European Union's "Financial Programming and Budget").[2]

In response, theoretical studies in the new economic geography (NEG) literature have sought to examine the impact of regional policy by using the Dixit-Stiglitz type of monopolistic competition of the standard new trade theory. However, firms in the models are atomistic and identical for simplicity's sake, in spite of governments and EU policy makers in the real world discussing how to attract high productivity firms, drawing attention to the reality of different firm productivities within sector and across sectors.

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[2] The EU budget totalled € 115,956 million for commitments and € 105,684 million for payments in 2005, which account for around 1 % of the GDP in the EU25. The second largest expenditure of the EU budget is spent on structural funds (heading 2) (€ 42,420 million in commitments and € 32,396 million in payment in 2005). One of the major spending in the structural funds is called "Objective 1" defined as "helping regions whose development is lagging behind catch up". It accounts for over 70 % of spending and is for infrastructure and production subsidies in less developed regions, which are defined as the regions of less than 75% of average GDP per capita in EU. From 2004 over 2006, the largest allocation in the Objective 1 in the Structural Funds were received by Poland and Hungary, Czech Republic, respectively € 8,275 million, € 1,995 million and €1,454 million. See more detail on http://ec.europa.eu/budget

Concerning firm productivity and heterogeneous-firms behaviour, the international trade literature has witnessed the recent emergence of results based on firm-level datasets, which has revealed a series of facts that cannot be explained by the standard new-trade theory (e.g. Bernard, Redding and Schott, 2004).[3] This new emphasis on firm-level empirical facts led a number of theorists to develop frameworks that allow for firm- level analysis. Utilising new advances from microeconomics that allow more detailed modelling of firms, a group of trade theorists has recently begun to develop trade models that account for the 'new' facts. One of the new theories stems from a brilliant contribution of Melitz (2003), which incorporates firm heterogeneity into general equilibrium trade models. His paper sought to model the impact of trade liberalisation on firm activity, focusing on the link between trade liberalisation and productivity. He used new advances from industrial economics by Hopenhayn (1992a,b) that allowed for more detailed modelling of firms such that he continues to view the firm as a profit function but allows for heterogeneity in firm's productivity. He found that trade liberalisation raises average firm productivity via a selection effect and via a productivity reallocation effect. Later, a group of trade theorists has further developed this research (e.g. Helpman, Melitz and Yeaple, 2004). However, this theoretical innovation has not yet been applied to questions of industrial location and new economic geography, nor has it been applied to regional policy analysis except for a few studies. Therefore, in order for the theoretical NEG literature in order to build further on the current strand in the theoretical NEG literature, and also so as to discuss the pros and cons of regional policies in Europe such as the Structural and Cohesion Funds, this paper models economic geography when firms are heterogeneous, and then studies the impact of the anti-agglomeration subsidy on industrial location.

1.1. *Literature review and this paper*

Heterogeneous-Firms Models

The heterogeneous-firms model literature, in which marginal costs are different across firms, has two branches. One strand, currently the main stream, is the heterogeneous-firms trade model (HFT) by Melitz (2003), Helpman, Melitz and Yeaple (2004), Falvey, Greenaway and Yu (2004), Melitz and Ottaviano (2005) and Yeaple (2005). Their trade models analyse the effect of trade liberalisation on productivity via a reallocation effect, allowing for free entry and exit. Trade liberalisation allows low productivity domestic firms to enter the export market and induces the least efficient domestic firms to exit from the local market. The other strand, which yet encompasses only a few studies, is the combination of HFT and NEG models (HFNEG) such as Baldwin and Okubo (2006a,b), Okubo and Rebeyrol (2006) and Okubo (2005). This strand is closest to our model.[4] Their models study the effect of firm heterogeneity on agglomeration forces and firm location via spatial sorting and selection in the framework of Martin and Rogers (1995) in the NEG literature.

The main findings in Baldwin and Okubo (2006a) are that the most efficient firms in a small region first relocate to the large market, and that a delocation subsidy allows the least efficient firms in the large region to move to the small region. Nevertheless, their analysis on relocation subsidies did not completely cover the complete set of the features. Since their focus was delocation tendencies and spatial sorting as a result of a per-firm delocation subsidy, this paper examines the equilibrium outcome and its features (e.g. agglomeration rent curves and Tomahawk diagrams) in a comparison

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[3] For example, firm differences within sectors may be more pronounced than differences between sector averages, and most firms –even in traded-goods sectors – do not export at all (Bernard and Jensen 1995, 1999a,b, 2001;Clerides, Lach and Tybout 1998, Aw, Chung, and Roberts 2000, Eaton, Kortum, and Kramarz 2004 and Bernard, Eaton, Jensen and Kortum 2003).

[4] In this strand, Baldwin and Okubo (2006b) extended Baldwin and Okubo (2006a) in the comparison with the HFT model of Melitz (2003). Okubo (2005) studied the impact of taxation on spatial sorting. Okubo and Rebeyrol (2006) studied the impact of firm heterogeneity on the home market effect.

with the homogeneous-firms model. In addition, since their analysis only features a per-firm (specific) subsidy, a proportional subsidy is discussed in this paper as contrast, in which the relocated firms receive a subsidy proportional to their profits. Further, since the resource of their subsidy was not specified (non-taxation finance), this paper takes into account taxation to implement the subsidy policy. Finally, as they did not conduct a welfare analysis, nor consider factor endowment differences, this paper deals with these issues in order to get a deeper insight and a better reflection of the European situation.

Subsidy and Industrial Location

Turning to the theoretical analysis on subsidy and economic geography, the impact of subsidies in a NEG setting with homogeneous firms was studied by Dupont and Martin (2003), which is closest to this paper with regards to viewpoints and analysis. They showed that the impact on location of such subsidies is stronger when trade is freer due to the home market magnification effect. They also study the income distribution effects, finding that although subsidies "constitute an official financial transfer from the rich to the poor region, they actually lead to an income transfer from the poor to the rich region" in certain cases. Further, different from Dupont and Martin (2003), Baldwin et al. (2003, Ch. 18) and Persson and Tabellini (2000) studied how regional policies are endogenously determined by voting and election.[5]

The remarkable importance of these theoretical works stems from clear evidence in contemporary Europe. As Braunerhjelm et al. (2000) mentioned, income inequality has increased at the regional level over the last two decades. To produce convergence of this inequality and promote economic development in the poor peripheral regions, most EU spending is channeled through structural spending programmes. In particular, the regional subsidy on manufacturing leads the periphery to attract firms. The effect of the European Structural Fund, aiming at geographical diversification of industrial location and convergence of the real income gap between core and periphery, has been studied empirically. Midelfart-Knarvik and Overman (2002) found that European Structural Funds have more or less significant effect on industry location, in particular for R&D intensive industries. De la Fuente (2002) found that the impact of the Structural Fund on the Spanish economy has been sizeable and eliminated 20 % of the initial gap in per-capita income with other advanced European nations.

Putting the present paper into this context, the aim of this paper is to study the impact of subsidy on firm location, like Dupont and Martin (2003), but whereas their paper hinges on the standard NEG framework--atomistic and identical productivity firms, we consider firm heterogeneity in a simple theoretical HFNEG model. This paper addresses the question of how subsidies affect firm location in a core-periphery structure, whether central or local government should take policy initiative and whether the subsidy can enhance welfare in the core as well as in the periphery. To answer this question, this paper models the impact of various relocation subsidies in the presence of firm heterogeneity, based on Baldwin and Okubo (2006a), in which firm relocation is allowed but entry and exit are prohibited. As a result, we find that 1) a proportional relocation subsidy attracts the high productivity firms to the periphery, while a specific relocation subsidy involves the least efficient firms to moving to the periphery first, 2) the periphery has an incentive to apply a local tax-financed proportional subsidy policy individually and 3) while subsidies cannot enhance world welfare when capital labour ratios are identical between regions, a subsidy in the asymmetric capital labour ratio regions can enhance world welfare.

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[5] Other related studies are subsidy competition in the game structure among governments and multinational firms in their location choice (Barros and Cabral, 2000; Fumagalli,2003; Albornoz and Corcos, 2005).

1.2. *Plan of the paper*

The paper is organised in five sections. The next section introduces the basic HFNEG model. Section 3 explores the equilibrium outcome for two different relocation subsidies, a specific subsidy and a proportional subsidy. Section 4 examines how subsidy rates are determined. We discuss a voting equilibrium, tax-financing of subsidies, socially optimal firm location, and the pros and cons of a relocation subsidy. Section 5 studies the case of asymmetric endowment ratios. The last section presents our concluding remarks.

2. THE BASIC MODEL

This section introduces a simple NEG framework in the presence of firm heterogeneity and interregional delocation of firms (capital movement). The model is based on Baldwin and Okubo (2006a) and so can be thought of as a marriage of the simplest NEG model akin to the footloose capital model of Martin and Rogers (1995) and the HFT model akin to Melitz (2003).

2.1. *The footloose capital model with heterogeneous firms*

As in the standard footloose capital (FC) model of Martin and Rogers (1995), our model works with two regions, two sectors and two factors of production. The regions – referred to as the North (core) and the South (periphery) – have identical technology, tastes, and openness to trade, but they differ in market size (the North has larger demand by convention). The two sectors are the M –sector (manufactures) and A-sector (a numeraire sector). The numeraire sector produces a homogenous good using labour subject to constant returns, perfect competition and costless trade. Then, the M-sector produces a continuum of varieties using labour as well as capital under increasing returns to scale, the Dixit-Stiglitz monopolistic competition and iceberg trade costs. Specifically, firms' marginal costs are flat with respect to the scale of production and international trade is subject to iceberg trade costs. We assume that each manufacturing firm needs one unit of capital as a fixed cost, with all variable costs involving only labour.

Capital can move between the North and the South, but labour is immobile. Since one unit of capital is employed per firm, capital mobility is tantamount to firm relocation. All capital is owned by labour so while capital is mobile, capital owners are not (this cuts out complications that would otherwise arise from backward linkages). In other words, each individual has an endowment of labour and capital.[6] For clarity's sake, each individual in the North and the South has a capital endowment consisting of a portfolio comprising a share of all firms' stocks. Thus, by sharing all firms' profits as stock holder, the capital reward per unit of capital is the average operating profits of all firms. Importantly, firms have heterogeneous unit labour requirements. In Melitz (2003) the distribution of these marginal costs is endogenised due to free entry and exit, but here we assume each region's distribution is fixed for simplicity.

Basic set up

The tastes of the representative consumer in each region are given by the Cobb-Douglas utility function:

$$U = C_M^{\mu} C_A^{1-\mu}, \quad C_M \equiv \left(\int_{i\in\Theta} c_i^{\,1-1/\sigma} di \right)^{1/(1-1/\sigma)}, \qquad 1 > \mu > 0, \quad \sigma > 1$$

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[6] In the basic model (sections 3 and 4), each individual has one unit of labour and capital due to K/L=K*/L*. In the asymmetric endowment ratio model in section 5, K/L>K*/L*, we assume that each individual has one unit of labour but more (in the North) or less (in the South) than one unit of capital. In both models, the per-unit capital reward is equal to the average profits.

where C_M represents consumption of the composite of M-sector varieties, C_A stands for consumption of the A-sector good, $\sigma>1$ is the constant elasticity of substitution between any two M-sector varieties and Θ is the set of all varieties produced. This set of varieties is pre-determined by endowments because each variety requires one unit of capital and the world capital stock is fixed, normalised to unity.

Firm-level heterogeneity in our model stems from differences in firm's marginal costs. Thus, although all the Dixit-Stiglitz varieties enter consumers' preferences symmetrically, the cost of producing each variety is different. Each firm employs one unit of capital so we naturally associate the firm-level differences with each firm's unit of capital.[7] Practically, what this means is that the unit labour requirements are variety-specific and thus firm-specific; we denote these as a_j for firm j. The distribution of firm-level efficiency is part of each region's endowment. The specific assumption is that the distribution of the a's is subject to the Pareto distribution, which cumulative density function is defined as:

$$\textbf{(1)} \qquad G[a] = \left(\frac{a^\rho}{a_0^{\ \rho}}\right), \qquad 1 \equiv a_0 \geq a \geq 0, \quad \rho \geq 1$$

Here a_0 is the scale parameter (highest possible marginal cost) and ρ is the shape parameter. Small ρ means more dispersed in 'a', that is, firms are more heterogeneous.[8] We can normalise a_0 to unity without loss of generality. To eliminate extraneous sources of relocation and trade, we assume G[a] is identical for the two regions. The cumulative density function of a's can be depicted in Figure 1. Note that the distribution in the North (the South) is nG[a] (n*G[a]). Since we initially consider full agglomeration in the North, the total mass of firms is 'n'=1 in the North and 'n*=0' in the South.[9]

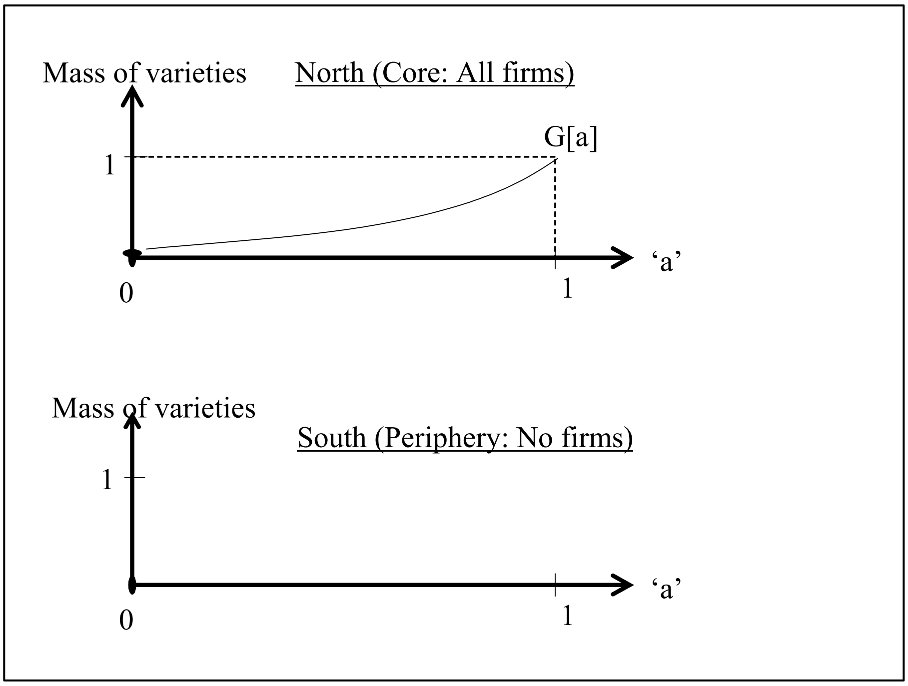

Figure 1: Endowed distribution of capital and the mass of firms (at initial equilibrium).

[7] It may help to think of capital as knowledge capital – a blueprint – that defines the production process and thus marginal cost

[8] At extreme, ρ=1 represents a uniform distribution.

[9] Since we take the range of varieties to be continuous, we speak of the 'mass' of firms with a particular marginal cost.

2.2. *Intermediate results*

Results for the numeraire sector in the two-sector models where one has costless trade and perfect competition are well appreciated. Constant returns, perfect competition and zero trade costs equalise nominal wage rates across regions. We choose units of the numeraire good such that p_A=1=w=w*. This means that all differences in M-firms' marginal costs are due to differences in their a's so we can refer to the a's as marginal cost without ambiguity.

The M- sector is marked by all the usual Dixit-Stiglitz results. Firms' prices are a constant mark-up of their marginal selling costs. In the local market, these marginal costs entail only production costs. The price in the export market includes the iceberg costs, t≥1, marked up by the constant Dixit-Stiglitz mark-up. Given the Cobb-Douglas utility function and portfolio assumption for capital returns, expenditure on manufactured goods is invariant to trade costs and firm location. Further we choose total units of labour and capital such that L^w=K^w=1. Since each individual has labour and capital, an individual's expenditure on all M-varieties is μE^w=μ (due to the normalisation, E^w=1).

Since firms/varieties are heterogeneous on the cost side and firms can locate in either region as well as sell in either market, we must be more precise than the usual Dixit-Stiglitz trade model about what is made where. The standard CES demand function for variety-j produced and sold in the North can be written as

$$c_j = (p_i)^{-\sigma}\frac{E}{\overline{P}}; \qquad \overline{P} \equiv \int_{i\in\Theta} p_i^{1-\sigma} di + \int_{h\in\Theta^*} \phi\, p_h^{1-\sigma} dh, \qquad \sigma > 1 \geq \phi \equiv t^{1-\sigma} \geq 0$$

where p_j is variety-j's producer price (which equals its consumer price since it is produced locally), and $\overline{P}$ is the (weighted) average of consumer prices of varieties sold in the North. The first term in the definition of $\overline{P}$ reflects the prices of goods that are produced in the North (and so bear no iceberg trade costs). The second term reflects the imported varieties whose producer prices are p_h; Θ and Θ* are the sets of goods produced in the North and the South, respectively. Note that the geometric weights are negative since σ>1, so $\overline{P}$ falls as the prices of individual varieties rise.

A parameter that plays a critical role in our paper is ϕ, which we refer to as the 'free-ness' (phi-ness) of trade. This is the usual iceberg trade cost factor, t≥1 (this tells us how much must be 'shipped' in order to sell one unit abroad) raised to the negative power 1-σ;

$$\phi \equiv t^{1-\sigma}, \quad t = 1 \Rightarrow \phi = 1, \quad t = \infty \Rightarrow \phi = 0$$

Note that when iceberg trade costs are prohibitive (t=∞), the freeness of trade is zero(ϕ=0), while when there are no iceberg trade costs (t=1), the freeness of trade is unity (ϕ=1).

2.3. *Initial Equilibrium—Full agglomeration*

Since agglomeration produces rent which holds firms in its own region, this rent makes various sorts of policies ineffective with regards to shifting firm location. To exclude the marginal impact of relocation subsidies and focus on the substantial impact of relocation policy on firm location, we start with full agglomeration. For this reason, we regard full agglomeration as initial equilibrium. In other words, we start to consider small trade costs so that all firms are in the North, i.e. above the sustain point: $\phi > \phi^{CP} = \frac{1-s_E}{s_E}$. Note that the mass of varieties are n=1 and n*=0.

Utility maximisation generates the familiar CES demand functions in the manufactures sector.[10] These, together with the standard Dixit-Stiglitz monopolistic competition assumptions on market structure, imply 'mill pricing' is optimal and operating profit earned by a typical firm in a typical market is 1/σ times firm-level revenue.[11] Accordingly, operating profit realised by a firm in the North is:

$$\textbf{(2)} \qquad \pi[a] = \frac{\mu E^W}{\sigma}\left(s_E \frac{\left(\frac{a}{1-1/\sigma}\right)^{1-\sigma}}{\int_i p_i^{1-\sigma} di} \; + \; (1-s_E)\frac{\phi\left(\frac{a}{1-1/\sigma}\right)^{1-\sigma}}{\int_i p_i^{*1-\sigma} di} \right)$$

The first term in the bracket is the value of firm-specific sales in the Northern market. Sales depend upon the firm-specific price, a/(1-1/σ), raised to 1-σ relative to the weighted average of the prices of all goods sold in the market (the integral is over all goods); E^W reflects the total market expenditure. The second term shows the firm's export sales, which is similar except the firm's price includes the iceberg trade cost raised to 1-σ and the denominator involves p*'s, the prices of all goods in the Southern market.

The features of **(2)** that matter for our results are 1) all firms earn a positive operating profit (this is the reward to capital, i.e. Ricardian rent), 2) the profitability of firms increases with firm-level efficiency, and 3) the most efficient firms, i.e. firms with low marginal costs, are the largest in the sense that they sell the most (their profits go to infinity).

The northern operating profit as a function the firm's 'a' can be written as:[12]

$$\textbf{(3)} \qquad \pi[a] = a^{1-\sigma}\left(\frac{s_E}{\overline{\Delta}} + \frac{\phi(1-s_E)}{\overline{\Delta}*}\right)\frac{\mu E^w}{K^w \sigma}; \quad s_E \equiv \frac{E}{E^w} > \frac{1}{2}$$

Here we have introduced s_E as shorthand for the North's share of world expenditure (we adopt the convention that the North is bigger so s_E >½), K^w is the world's endowment of capital, and the $\overline{\Delta}$'s are the denominators of the North and the South CES demand functions in initial equilibrium, i.e. full agglomeration in the North ($\overline{\Delta}$ is a mnemonic for denominator):

$$\overline{\Delta} \equiv \int_0^1 a^{1-\sigma} dG[a];$$

$$\overline{\Delta}* \equiv \phi\int_0^1 a^{1-\sigma} dG[a];$$

Since we consider all firms concentrating in the North as initial equilibrium, we have:[13]

$$\textbf{(4)} \qquad \overline{\Delta} = \lambda; \quad \overline{\Delta}* = \lambda\phi; \quad \lambda \equiv \frac{\rho}{1-\sigma+\rho} > 0, \;\; \rho > 1; \;\; \phi > \phi^{CP} \equiv \frac{1-s_E}{s_E}$$

where we assume 1-σ+ρ>0 (which ensures the integrals converge) and $\phi > \phi^{CP}$, i.e. smaller trade costs than the sustain point, together the full agglomeration conditions.

2.4. ***Relocation subsidy***

Relocation subsidy is employed so as to induce some firms to relocate to the periphery. Our model

[10] Individual demand for a typical variety j is $c(j)=p(j)^{-\sigma}\mu/\overline{P}$.

[11] A typical first order condition is p(1-1/σ)=wa; rearranging, the operating profit, (p-wa)c, equals pc/σ.

[12] This simplification uses mill pricing and cancels the (1-1/σ) terms.

[13] Since firms are atomistic, the first firm to move has no impact on the Δ's.

provides equilibrium features with regard to two sorts of relocation subsidy, specific subsidies and proportional subsidies. Using **(3)**, a specific subsidy, *S*, is written in the operating profit in the South as

(5) $$\pi^*[a] = a^{1-\sigma}\left(\frac{\phi s_E}{\Delta} + \frac{1-s_E}{\Delta^*}\right)\frac{\mu E^w}{K^w \sigma} + S; \quad s_E = \frac{E}{E^w} > 0.5$$

By contrast, a proportional relocation subsidy, *u*, is proportional to each firm's operating profit, so that, using **(3)**, operating profit as a function of the relocated firm's 'a' can be written as:

(6) $$\pi^*[a] = a^{1-\sigma}\left(\frac{\phi s_E}{\Delta} + \frac{1-s_E}{\Delta^*}\right)\frac{\mu E^w}{K^w \sigma}(1+u); \quad s_E = \frac{E}{E^w} > 0.5$$

where *u* denotes the rates of a proportional relocation subsidy. Note that the subsidy, which adds to the profits of the relocated firms, never affects the mass of varieties and prices in Δ and Δ*.

2.5. *A specific subsidy and its delocation tendency*

Starting from a core-periphery structure, the northern base firms relocated to the South are subsidised. Each firm, moving to periphery, receives a per-firm fixed amount of subsidy, *S*. The change in operating profit for an atomistic firm moving from the north to the smaller south would be:

(7)

$$\pi[a] - \pi^*[a] = a^{1-\sigma}\frac{\mu E^w(1-\phi)}{\sigma}\left(\frac{s_E}{\bar{\Delta}} - \frac{1-s_E}{\bar{\Delta}^*}\right) - S = a^{1-\sigma}\frac{\mu(1-\phi)}{\lambda\sigma}\left(s_E - \frac{1-s_E}{\phi}\right) - S; \quad s_E > 0.5, \phi > \phi^{CP}$$

for a firm with the marginal cost of 'a'. For this, the first term would be positive due to $\phi > \phi^{CP}$ and thus equation **(7)** can only be negative for a substantial level of the subsidy, *S*. Importantly, the loss from relocation, ignoring the subsidy, is decreasing in the firm's marginal cost parameter 'a' (Figure 2). The most efficient firms find location in the big region most profitable, so they are also the ones that would sacrifice the most by relocating to the small region (the South). If we started with a very small subsidy and increased it, the first firms to relocate to the small region (the South) would be the least efficient firms.

The minimum effective subsidy, which can induce only the least efficient firms (a=1) to move to the small region, can be derived analytically in finite range. However, the maximum subsidy, which can induce all firms to move to the small region, is infinite, because the profit gap for the most efficient firms,(a=0) goes to infinity.

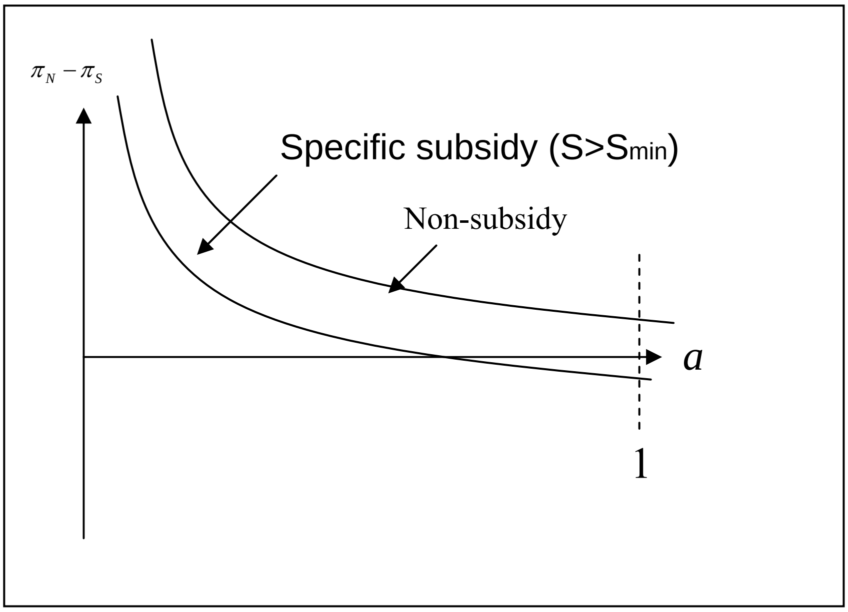

Figure 2: Delocation tendencies with a specific subsidy.

Result 1 (Baldwin and Okubo, 2006a): The first firms to respond to a specific subsidy will be the least efficient firms. The minimum effective specific subsidy, $S^{\min}$, is $\frac{2\mu(1-\phi)((1+\phi)s_E-1)}{\lambda\sigma\phi}$. To induce all firms to move to the small region, i.e. the maximum rates, the subsidy would have to be infinite.

Note that $S^{\min}$ is a function of demand size (s_E), demand structure (σ), and firm heterogeneity (ρ).

2.6. *A proportional subsidy and its delocation tendency*

Next, we study a proportional subsidy, which demonstrates the opposite result of a specific subsidy. This subsidy is proportional to the pure profits of firms in the South. Starting with all firms in the North, the change in operating profit for an atomistic firm moving from the North to the small south would be:[14]

(8)

$$\pi[a]-\pi^*[a]=a^{1-\sigma}\left(\frac{s_E}{\bar{\Delta}}+\phi\frac{(1-s_E)}{\bar{\Delta}^*}-\left(\phi\frac{s_E}{\bar{\Delta}}+\frac{(1-s_E)}{\bar{\Delta}^*}\right)(1+u)\right)\frac{\mu E^w}{K^w\sigma}=\frac{a^{1-\sigma}\mu}{\sigma\lambda}\left(1-\left(\phi s_E+\frac{1-s_E}{\phi}\right)(1+u)\right);$$
$$s_E>0.5;\phi>\phi^{CP}$$

As long as the above equation is positive, no firms have incentive to move to the South. Above a certain level of subsidy, $u^{\min}$, in contrast, the above equation switches from positive to negative. As shown in Figure 3, the most efficient firms are those most likely to relocate to the small region. Note that the minimum subsidy, $u^{\min}$, such that only the most efficient firms move to the South, is satisfied as $1-\left(\phi s_E+\frac{1-s_E}{\phi}\right)(1+u^{\min})=0$ and the subsidy rates to induce all firms to move to the

[14] This simplification uses mill pricing and cancels the (1-1/σ) terms. From **(4)**, $\Delta^*=\phi\,\Delta$ is derived.

South are given as $u^{\max} = \frac{s_E}{\phi} + \phi(1 - s_E) - 1$.

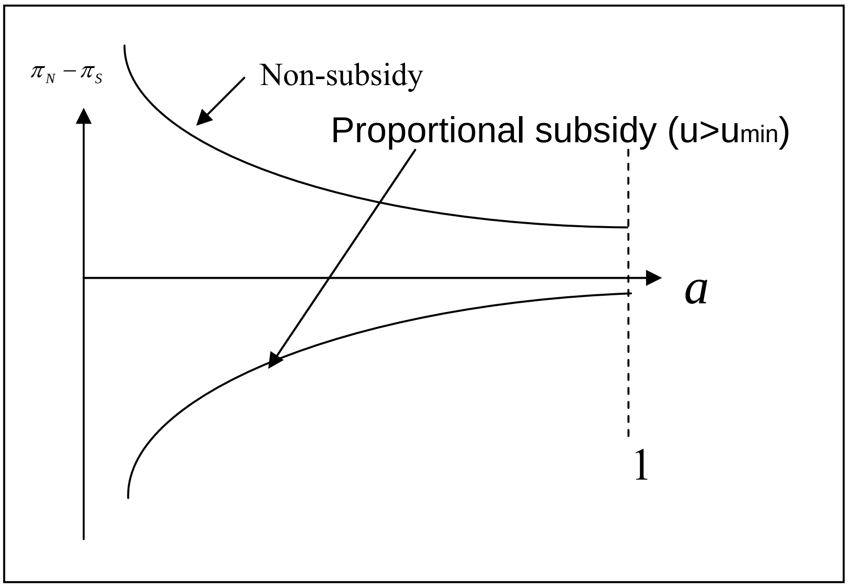

Figure 3: Delocation tendencies with a proportional subsidy.

Result 2: The first firms to respond to a proportional relocation subsidy will be the most efficient firms. The minimum subsidy rate is $u^{\min} = \frac{1}{s_E\phi + \frac{1-s_E}{\phi}} - 1$, which can induce the most efficient firms to move to the small region. The maximum rate is $u^{\max} = \frac{s_E}{\phi} + \phi(1 - s_E) - 1$ which stimulates all firms to move to the small region.

Note that in contrast to what is observed with a specific subsidy, minimum and maximum rates are independent of firm heterogeneity (λ, ρ), and a maximum rate is finite.[15]

2.7. *The basic mechanism*

Without relocation subsidies, as the standard FC model of Martin and Rogers (1995) and HFNEG model of Baldwin and Okubo(2006a) have concluded, there is no possibility for capital movement from core to periphery when $\phi > \phi^{CP} = \frac{1-s_E}{s_E}$. This is caused by small market size and no cost advantage in the periphery (the same production costs as in the core but small demand in the periphery).

[15] The minimum rate of *u* is always positive, nor negative (tax). From the full agglomeration condition in the initial equilibrium (non-subsidy): $\pi[a] - \pi^*[a] = \frac{\mu}{\sigma}\left(1 - \phi s_E - \frac{1-s_E}{\phi}\right) > 0$, $1 - (\phi s_E + \frac{1-s_E}{\phi}) > 0$ can be derived. Thus, the minimum subsidy is thus: $u^{\min} = \frac{1}{s_E\phi + \frac{1-s_E}{\phi}} - 1 > 0$.

However, once subsidies are introduced, things change: A specific subsidy attracts only the least efficient firms in the core, because the subsidy is fixed per-firm and independent of firm productivity, and meanwhile the productivity difference gives rise to different pure profits. The fixed amount of subsidy is a small portion in the profits of the high productivity firms, while it is a relatively large portion in the low productivity firms' profits. Therefore, the low productivity firms are likely to delocate to the periphery in order to receive the subsidy to enlarge their tiny profits.

On the other hand, a proportional type subsidy yields in a different mechanism. The subsidy is not uniform, but proportional to firm size. High productivity firms can receive more subsidy than the low productivity ones (in absolute value), implying that the most efficient ones can obtain the largest subsidy, and vice versa. Since the most efficient firms are most sensitive in their location and profits, they are the first one to relocate to the South benefiting from the largest amount of subsidy.

2.8. *Agglomeration rent curves*

Following the standard NEG literature, agglomeration rent can be specified as **(7)** and **(8)**. Firm heterogeneity, higher λ (lower ρ), reduces agglomeration rents in both subsidies. Figure 4 plots agglomeration rents in terms of ϕ, given a certain level of subsidy (given *u* and *S*). The rise of subsidy rates reduces agglomeration rent and shifts its curve downwards. Different from the standard model, the curve has sustain and break points in trade liberalisation: As trade gets freer, agglomeration rents first rise and then fall and finally the rent becomes negative due to the subsidy. What this means is that with very small trade costs, full agglomeration becomes unstable again and diversification is likely to occur. At the extreme, with free trade ($\phi = 1$), negative agglomeration rents correspond with the received (negative) subsidy, respectively $-\frac{\mu}{\sigma\lambda}a^{1-\sigma}u$ for a proportional subsidy and -*S* for a per-firm subsidy. This implies that the impact of subsidy on the agglomeration gap increases as trade gets freer and thus a given level of subsidy has more effect during trade liberalisation and deeper integration, until finally in free trade the full amount of subsidy passes through as agglomeration rents.

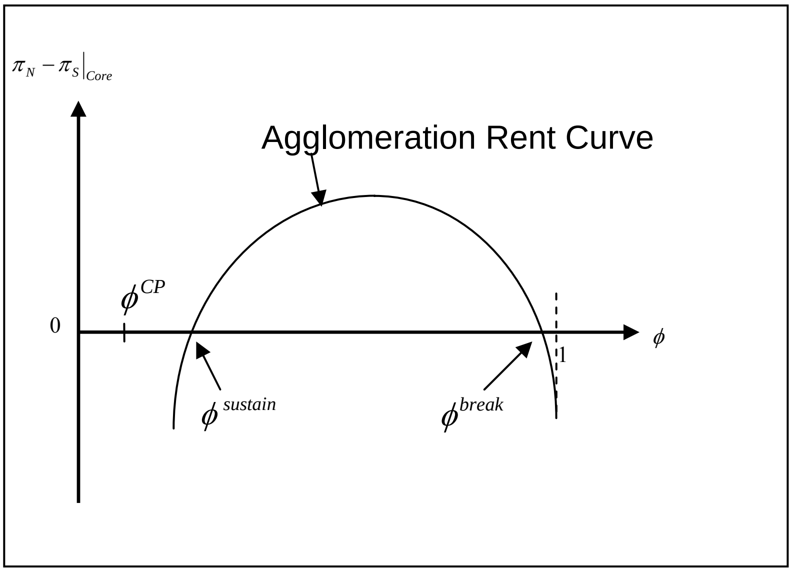

Figure 4: Agglomeration rent curve with a delocation subsidy.

Sustain and break points are analytically solved as

(9) $$\phi^S = \frac{1-\sqrt{1-4s_E(1-s_E+S\frac{\sigma\lambda}{\mu})}}{2s_E};\ \phi^B = \frac{1+\sqrt{1-4s_E(1-s_E+S\frac{\sigma\lambda}{\mu})}}{2s_E}$$

for a specific subsidy and

(10) $$\phi^S = \frac{\frac{1}{1+u}-\sqrt{\left(\frac{1}{1+u}\right)^2-4s_E(1-s_E)}}{2s_E};\ \phi^B = \frac{\frac{1}{1+u}+\sqrt{\left(\frac{1}{1+u}\right)^2-4s_E(1-s_E)}}{2s_E}$$

for a proportional subsidy.[16] As seen in **(9)** and **(10)**, while not affecting the sustain and break points for a proportional subsidy, firm heterogeneity, λ, affects them for a specific subsidy. In case of a specific subsidy, as firms are more heterogeneous (higher λ, lower ρ), the positive range of agglomeration rent curve (between sustain and break points) shrinks: the break point falls and the sustain point rises in λ. Overall, the agglomeration rents for specific subsidies also reduce as firms are heterogeneous. By contrast, in case of a proportional subsidy, since more firm heterogeneity reduces operating profits, the amount of the subsidy reduces proportionally. Due to this proportionality of subsidy and profits, firm heterogeneity never shifts the sustain and break points, in spite of reducing rents.

Result 3: An increased subsidy reduces agglomeration rents. The hump-shaped rent curve shifts downwards. There exist not only sustain points, $\phi^{Sustain} > \phi^{CP}$, but also break points, $\phi^{Break} < 1$, for both types of subsidies.

Result 4: For a specific subsidy, more firm heterogeneity lowers the break point and raises the sustain point. By contrast, for a proportional subsidy, firm heterogeneity affects neither points.

3. LOCATIONAL EQUILIBRIUM

3.1. *Specific and proportional subsidies*

Based on delocation tendencies (Section 2.5 and 2.6), the locational equilibrium for a tax-financed subsidy is explored. As seen in Result 1, the least efficient firms are likely to relocate to the South in case of a specific subsidy. To work out the precise relationship between the subsidy *S* and the cut-off marginal cost, we note that if all the least efficient firms with marginal costs in excess of a_S move to the south, the Δ's will be:

(11) $$\pi[a]-\pi^*[a] = a^{1-\sigma}\frac{\mu E^w(1-\phi)}{K^W\sigma}\left(\frac{s_E}{\Delta}-\frac{1-s_E}{\Delta^*}\right)-S;\quad s_E>0.5, \phi>\phi^{CP}$$
$$\Delta = \lambda\left(a_S^{1-\sigma+\rho}+\phi(1-a_S^{1-\sigma+\rho})\right),\qquad \Delta^* = \lambda\left(\phi a_S^{1-\sigma+\rho}+1-a_S^{1-\sigma+\rho}\right)$$

where a_S is the cut-off level of efficiency above which firms move.

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[16] Note that when *u* and *S* go to zero (non-subsidy), the sustain and break points converge to the ones in the standard footloose capital model, i.e. $\phi^s = \phi^{cp} = (1-s_E)/s_E$ and $\phi^B = 1$. For a proportional subsidy, the sustain and break points in the homogeneous-firms model are the same as in the heterogeneous-firms model, but the sustain and break points for a specific subsidy in the homogeneous-firms model are expressed as $\phi^S = \frac{1-\sqrt{1-4s_E(1-s_E+S\frac{\sigma}{\mu})}}{2s_E};\ \phi^B = \frac{1+\sqrt{1-4s_E(1-s_E+S\frac{\sigma}{\mu})}}{2s_E}$, which are different from the heterogeneous firms model. This discrepancy between homogeneous and heterogeneous cases can be found in Figure 5, in which the sustain and break points with a specific subsidy in the homogenous-firms model are respectively larger and smaller than in the heterogeneous-firms model.

The locational equilibrium is derived by solving **(11)**. a_S cannot be solved analytically because 1-σ+ρ and 1-σ are different values of non-integer powers, although it has a unique and positive solution. Also, a negative relationship can be found between *S* and a_S : the rise of the subsidy decreases a_S , which means that the increased subsidy promotes firm delocation.

Next, following Result 2 the Δs in the equilibrium with a proportional subsidy are written as,

$$\textbf{(12)}\qquad \pi[a]-\pi^*[a]=a^{1-\sigma}\frac{\mu E^w}{K^w\sigma}\left(\frac{s_E}{\Delta}+\phi\frac{(1-s_E)}{\Delta^*}-\left(\phi\frac{s_E}{\Delta}+\frac{(1-s_E)}{\Delta^*}\right)(1+u)\right);\quad s_E>0.5;\phi>\phi^{CP}$$

$$\Delta=\lambda\left(\phi a_P{}^{1-\sigma+\rho}+(1-a_P{}^{1-\sigma+\rho})\right),\qquad \Delta^*=\lambda\left(a_P{}^{1-\sigma+\rho}+\phi(1-a_P{}^{1-\sigma+\rho})\right)$$

where a_P is the cut-off level of efficiency below which firms move('P' is a mnemonic for proportional subsidy.). Solving **(12),** a_P can be solved as a locational equilibrium. Different from a specific subsidy, a_P can be solved analytically in the equilibrium:

$$\textbf{(13)}\quad a_P=\left(\frac{1-\phi\chi}{(1-\phi)(1+\chi)}\right)^{1/(1-\sigma+\rho)};\quad \chi\equiv\frac{(1-\phi(1+u))}{(1+u-\phi)}\frac{s_E}{1-s_E}$$

where *u* is positively correlated with a_P . This means that a higher rate of subsidy promotes firm delocation.

Result 5: (Analytical solvability) For a proportional subsidy, the equilibrium (cutoff level) is analytically solvable although this is not the case for a specific subsidy.

This analytical solvability tells us that analytically unsolvable cutoff levels in HFNEG models (Baldwin and Okubo, 2006a,b; Okubo and Rebeyrol, 2006), due to different non-integer powers, are caused by per-firm costs and subsidies, and thus if these costs and subsidies would be proportional to their operating profits, this problem could be resolved.

3.2. *Locational equilibrium and a comparison to the homogeneous-firms model*

The locational equilibrium can be depicted diagrammatically. In order to highlight the impact of firm heterogeneity, we compare the locational equilibrium with the homogeneous-firms model, i.e. the standard FC model, (setting marginal costs, ´a', to unity for all firms without loss of generality) with both types of subsidies.

First of all, for a given (finite) specific subsidy, Figure 5 (the so-called Tomahawk diagram) plots the production share (s_P) and freeness of trade (ϕ). The locus in the heterogeneous case reaches s_P =0 just at ϕ=1. This is consistent to Result 1: the subsidy rate necessary to mobilise all firms should be infinite; to put it the other way round, the most efficient firms could move to the South only at ϕ=1 with a finite subsidy rate. By contrast, in the homogeneous-firms model, the trade costs required to induce all firms to move to the South are ϕ<1. What this means is that firm heterogeneity leads efficient firms to bind more to the northern location. To put it briefly, the highest productivity firms' profits are infinitely large, yet subsidy rates are fixed irrespective of their productivities, and thus these firms prefer to stay in the North at ϕ<1.

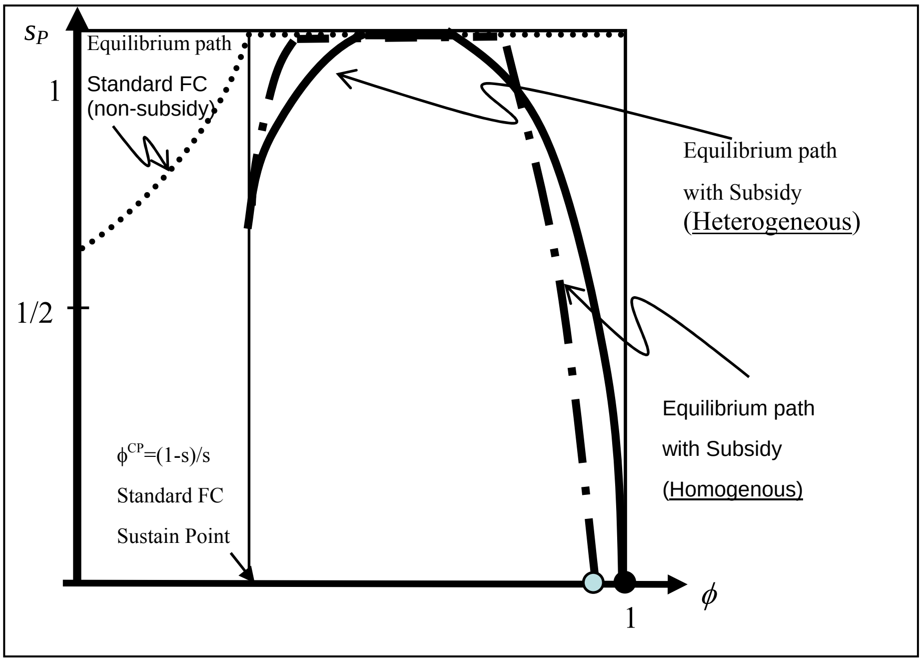

Figure 5: Tomahawk diagram with specific subsidies.

Regarding the proportional subsidy, contrasting results can be found. As seen in Figure 6, firm heterogeneity just dampens the delocation process, arriving at the same break and sustain points as in the homogeneous-firms model. This is due to the proportionality of subsidy and profits, correspondent to Result 2. Since all the delocated firms have high productivity in the heterogeneous firms model, more severe competition in the South dampens the delocation process with firm heterogeneity.

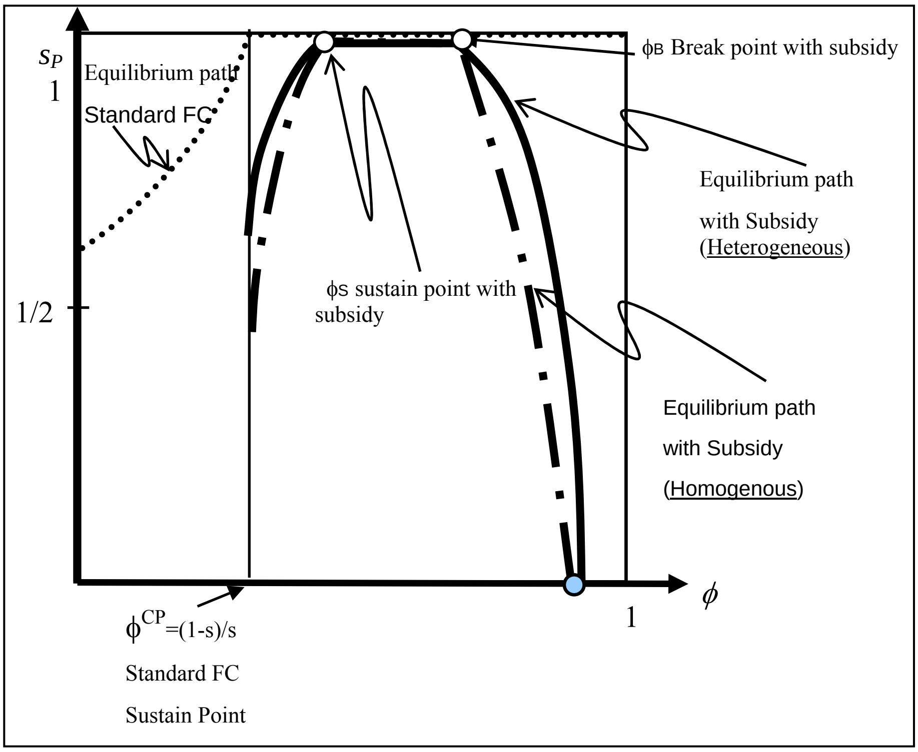

Figure 6: Tomahawk diagram with proportional subsidies.

3.3. ***Welfare analysis—specific vs. proportional Subsidies***

Based on the equilibrium discussion in the last section, we conduct a welfare analysis of the heterogeneous-firms model. The measurement of welfare uses the per-capita indirect utility function,

(14) $$v = \ln(E - T) + \frac{\mu}{\sigma - 1}\ln\Delta \quad \text{and} \quad v^* = \ln(E - T^*) + \frac{\mu}{\sigma - 1}\ln\Delta^*$$

where E is a per-capita income and T and T* are per-capita tax. Each individual owns one unit of labour and one unit of capital. E=w+ $\bar{\pi}$ =1+ $\bar{\pi}$, where $\bar{\pi}$ is reward per unit of capital, which is average operating profit of all firms due to portfolio assumption.[17] Here, we assume that the subsidy is financed by per-capita taxation, and the government budget is balanced. In this section, the taxation, *T*, is assumed to be equally levied on all people in the core and periphery by a central government (global tax-financing scheme). We notice that the tax is refunded as subsidy to relocated firms and then finally returned to all people equally as capital rewards due to the portfolio assumption. Since the increase of the capital reward due to the subsidy is equally distributed over all people, the tax payment and the delocation subsidy keep income constant and just affect firm relocation and thus Δ.[18] Figure 7 plots welfares in both subsidy schemes— specific and proportional subsidies scaled to the same total amount of subsidy. Given the same total amount of subsidy, the two schemes lead to different results with respect to per-capita welfare between the two regions. Note that the vertical axis represents welfare, *v*[u,S] in the North, *v**[u,S] in the South, and the horizontal axis depicts the fixed subsidy, *S*, or the proportional one, *u*. Without subsidy, i.e. *S*=0 and *u*=0, the northern welfare always exceeds the southern one, *v*[0]>*v**[0], where $v[0] = \left(\ln(1 + \frac{\mu}{\sigma}) + \frac{\mu}{\sigma - 1}\ln(\lambda) \right)$ and $v^*[0] = \left(\ln(1 + \frac{\mu}{\sigma}) + \frac{\mu}{\sigma - 1}\ln(\phi\lambda) \right)$.[19] Increased subsidy moves more firms to the periphery and thus reduces the northern per-capita welfare while increasing southern per-capita welfare by changing Δ and Δ*.[20]

With a certain amount of *S'*, the dotted curves in Figure 7 stand for higher welfare in the North and lower welfare in the South for a specific subsidy. Given a fixed total amount of subsidy, if the government switches to a proportional subsidy (the solid curves in Figure 7), *u'*, this results in equalising welfare in both regions. To equalise welfare between the core and periphery, the proportional subsidy requires *u'*(=*S'*) but the specific subsidy needs *S''*(>*S'*). At the extreme, all firms move to the South with the highest subsidy ($u^{\max}$), at which the periphery's welfare switches to the initial welfare in the core, and vice versa. By contrast, the maximum specific subsidy is infinite, corresponding to Result 1.

Intuitively, this discrepancy of subsidy policies stems from different relocation tendencies due to firm heterogeneity: a proportional subsidy can induce the most efficient firms to relocate to the periphery, rapidly increasing the southern welfare. The most efficient firms' relocation as a result of a proportional subsidy even quicker increases Δ* and can equalise welfare for a smaller amount of subsidy.

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[17] In the asymmetric endowment model, as will be shown in Section 5, the return per unit of capital is identical across regions, but the return per capita is different across regions, because the units of capital owned by individual are different across regions due to asymmetric endowment.

[18] If tax is imposed only on southern residents (local tax-financing scheme), as discussed later, the subsidy introduces a discrepancy in disposable income between regions.

[19] This is correspondent to full agglomeration in the standard footloose capital model (non-subsidy equilibrium). Note that w=w*=1, $\bar{\pi} = \mu/\sigma$, $\Delta = \lambda$ and $\Delta^* = \lambda\phi$. Tax payment, *T*, and subsidy are null due to non-subsidy.

[20] Notice that global tax financing scheme has no effect on disposable income. Thus, a higher subsidy never reduces disposable income and just changes firm allocation.

Result 6: A proportional subsidy can equalise welfare for a smaller total amount of subsidy. A proportional subsidy is more efficient than a specific subsidy in the sense of smaller subsidy equalising per-capita welfare.

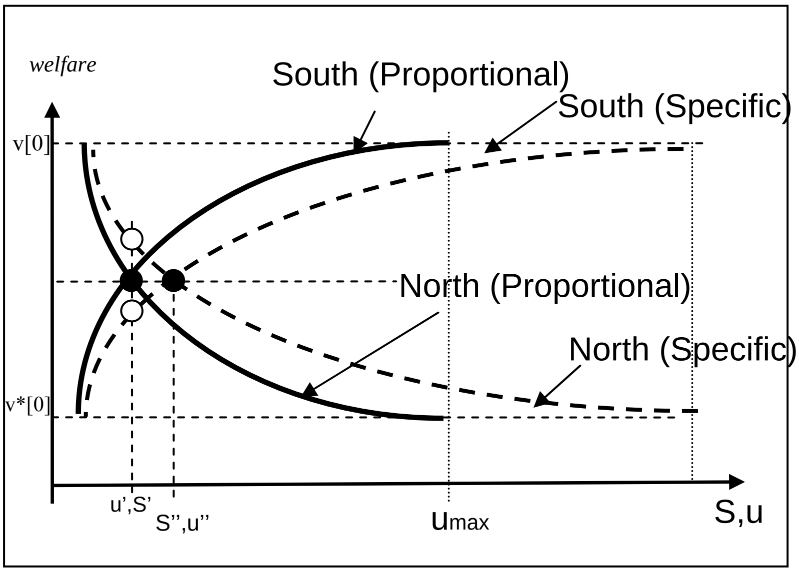

Figure 7: Per-capita welfare and subsidy rates.

Before the next discussion, we address the question of which subsidy would be chosen, a proportional or a specific one. Clearly, the proportional subsidy should be adopted. One reason is efficiency, as concluded in Result 6. A proportional subsidy could attract the most efficient firms. This could enhance the southern welfare more at smaller subsidy rates and could minimise firm relocation (efficient but a smaller number of relocated firms). The other reason is productivity effect. In reality, the South definitely attempts to attract the high productivity firms rather than low productivity firms, taking into account productivity gain from the subsidies. Thus, a proportional subsidy is beneficial in attracting high productivity firms. For these reasons, a proportional subsidy is preferred. Hence, our analysis below focuses on the case of proportional subsidies.[21]

4. GLOBAL VS LOCAL TAX-FINANCING SCHEME

4.1. *Voting equilibrium and global tax-financed subsidy*

We analyse here how the electoral system has an impact on the spatial equilibrium in a core-periphery structure. Since a central government has authority to implement subsidy policy, it is thought of as coordinator between core and periphery interests, attempting to reduce the wealth gap. Then, voting by all residents in the core and periphery perfectly reflects subsidy policy-making of the central government: Based on each individual's welfare, v and v^*, voting determines the rate of subsidy. Each individual has equal voting right. Taxation is global, i.e. per capita tax in both regions, and the government budget is balanced:

(15) $$T = T^* = \frac{\mu\lambda}{\sigma}\left(\phi\frac{s_E}{\Delta} + \frac{1-s_E}{\Delta^*}\right)a_P{}^{1-\sigma+\rho}u$$

[21] In section 4, even in case of a specific subsidy, the results can be kept as for the proportional subsidy, except for the impact of firm heterogeneity on the optimal subsidy for the South (footnote 23 in section 4.3). In particular, the southern welfare curve is hump-shaped in both sorts of subsidy, which is a crucial result. See Appendices 1 and 2.

The northern residents prefer zero-subsidy to keep all firms in the North without taxation, whereas the southern people prefer positive rates of relocation subsidy to attract firms to the South along with taxation of all people. Note that the North is larger than the South in population ($s_L = s_K = s_E > 0.5$). Since the zero-subsidy outcome preferred by the northern large population is dominant due to the majority rule, the subsidy would be zero in the voting equilibrium. Accordingly, voting could not resolve the gap of per-capita welfare across regions.

Result 7: When voting determines subsidy rates associated with per-capita taxation in both regions (global tax-financing scheme), the non-subsidy outcome is an equilibrium, where core residents' preference, full agglomeration, is dominant.

4.2. *Local tax-financed subsidy*

As seen in section 4.1, the voting mechanism rejects any relocation subsidy in a centralised state due to the majority rule. Now, suppose a decentralised nation without central government and without voting. Then, if the periphery has authority to implement the subsidy policy without voting mechanism, it has an incentive to use subsidy financed by southern residents (i.e. local tax-financed subsidy). Accordingly, a non-subsidy outcome could be unstable.

Figure 8 plots per-capita welfares, in which the southern welfare is hump-shaped due to the tax collection ("South (Local)" in the figure), while the northern welfare, "North (Local)", has a similar shape as before. The hump-shape southern welfare induces the South to set up the rates of subsidy such as to maximise the welfare, u^*.[22] In the South, higher rates of subsidy attract more firms but at the same time increase southern tax payment, T^*, which leads to a reduction in disposable income. In the last section, the global tax financing scheme, in which tax is equally levied in both regions and subsidy is distributed equally via capital returns, has no income effect. However, the local tax financing scheme is associated with taxation <u>only in the periphery</u> and the subsidy is <u>refunded to both regions' people</u> through capital rewards. In the North, without paying tax, T =0, the North receives a part of the subsidy through capital rewards of southern firms. The per capita tax, levied only in the South, is

(16) $$T^* = \frac{\mu\lambda}{\sigma}\left(\phi\frac{s_E}{\Delta} + \frac{1-s_E}{\Delta^*}\right)\frac{a_P{}^{1-\sigma+\rho}u}{1-s_L}.$$

where the total amount of subsidy is divided by the southern population ($1-s_L$) ($=(1-s_E)$).

Thus, a local tax-financed subsidy plays a role in income transfer from periphery to core: Higher subsidy reduces disposal income in the periphery (South) and increases income in the core (North). This income transfer raises northern welfare and decreases welfare in the South. Tax payment puts a heavy burden on southern people, and finally southern welfare declines for sufficiently high level of subsidy.

[22] See Appendix 2 for analytical proof of the hump-shape curves.

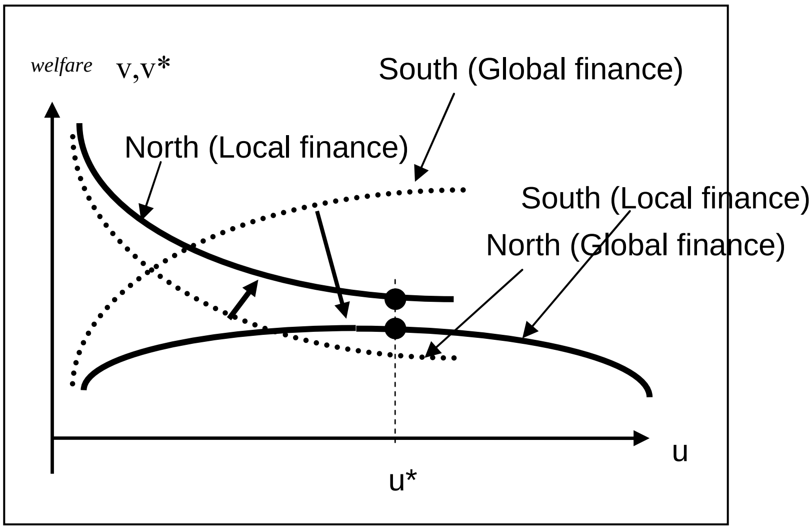

Figure 8: Global vs. local tax-financing scheme and per-capita welfare.

Result 8: In the case of decentralised taxation, the periphery will choose to independently subsidise firms by locally–financed taxes so as to attract the northern firms.

Next, we allow for both regions' subsidy to be financed by each region, and then a 'subsidy war' could occur to attract high productivity firms of the other region by each local government's local tax-financed subsidy. Once the South subsidises firms to attract to the South, the North attempts to subsidise them to keep full agglomeration as retaliation. Since the North has a wider tax base due to the higher share of population in the North, it always wins. Regardless of the same wage and capital returns per capita (average returns), tax per capita in the South always needs to be larger than in the North at the same level of subsidy, and thus in the end of the subsidy war southern individual's (gross) income would be equal to the per-capita tax, and the South would not be able to increase subsidy rates any more. Hence, the North wins the war.

Result 9: In a decentralised state, once a subsidy war occurs, initiated by a southern locally tax-financed subsidy, the North always wins and the South always loses. The Nash equilibrium is full agglomeration in the core with no subsidies in both regions (*u*=0) or the subsidy rates such that the total amount of subsidy is equal to southern total income.

4.3. *Trade costs, market size in optimal local tax-financing scheme*

There exist optimal southern subsidy rates, u^*, so as to maximise southern welfare in the local tax financing scheme. Now, we study how the optimal subsidy rates change when trade costs and expenditure shares change. The aim is to discuss the impact of trade liberalisation on southern subsidy rates and the impact on southern growth.

Trade liberalisation increases southern welfare for any u^*. Figure 9 shows a negative correlation between $u^*[\phi]$ and ϕ, suggesting that the South attracts more firms as ϕ decreases (this increases southern welfare), $\frac{\partial u^*[\phi]}{\partial \phi} < 0$. This result is similar to Dupont and Martin (2006), where the mechanism is such that deeper economic integration causes stronger impact of subsidy on firm relocation due to the home market magnification effect.

Turning to market size, ceteris paribus, an increase in s_E boosts $u^*[s_E]$ (Figure 10), $\frac{\partial u^*[s_E]}{\partial s_E} > 0$. As the northern market is larger, a higher subsidy is necessary to attract firms to the South. The larger market size of the core is not likely to switch to the periphery in firm location. Finally, while a higher degree of firm heterogeneity increases welfare in the South at *u**, firm heterogeneity never affects *u**, $\frac{\partial u^*[\rho]}{\partial \rho} = 0$ due to the neutrality of $a^{1-\sigma+\rho}$ from firm heterogeneity.[23]

Result 10: Trade costs reduction, ceteris paribus, strengthens the impact of subsidy and consequently decreases the optimal subsidy rate. Increased northern market expenditure (market size) share, ceteris paribus, increases an optimal subsidy by weakening the impact of subsidy. However, firm heterogeneity never affects the optimal subsidy rate for a proportional subsidy.

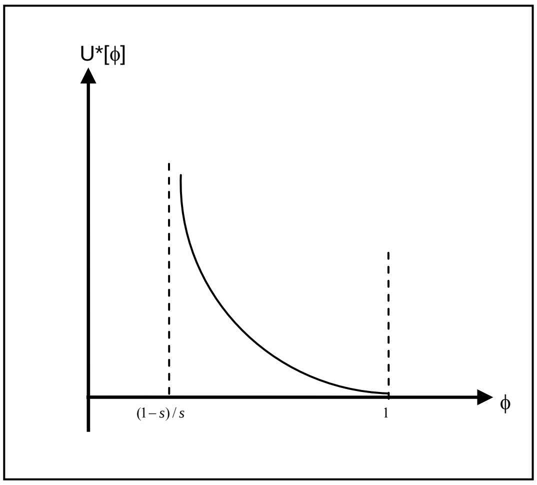

Figure 9: Southern optimal subsidy and trade costs.

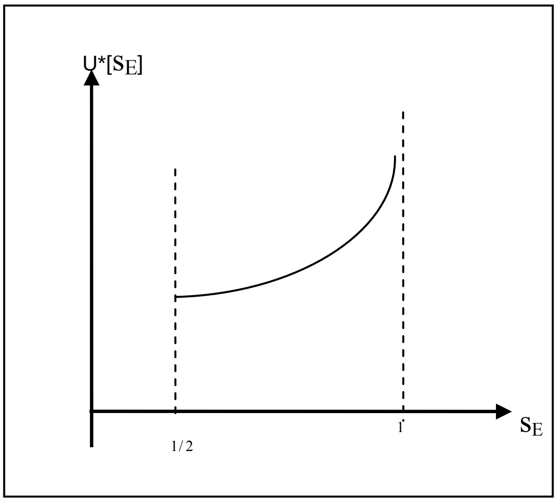

Figure 10: Southern optimal subsidy and market shares.

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[23] Given u*, $a_P^{1-\sigma+\rho}$ is independent of firm heterogeneity and dependent on s_E and ϕ, as shown in **(13)**. Hence, ρ is neutral in *u*. For a specific subsidy, as shown in Appendix 1, a specific case always has a similar humped-shape southern welfare curve under a certain sufficient condition. We can get the same effect of market size and trade costs on an optimal specific subsidy. However, different from the proportional case, we get $\frac{\partial S^*[\rho]}{\partial \rho} > 0$. Clearly, *S*, ceteris paribus, increases in ρ in equilibrium as seen in **(11)**.

4.4. *Social welfare and socially optimal subsidy*

Finally, we discuss optimal welfare in the world, when the South applies a local tax-financed subsidy policy. The northern/ southern welfare and world welfare are respectively defined as $V^N = s_L v$, $V^S = (1-s_L)v^*$. Our interest is world welfare, V^W, with respect to subsidy: $V^W = s_L v + (1-s_L)v^*$.[24] In V^W, as previously discussed, total income in the world is invariant to subsidy policy, because any local/global tax-financed subsidy is refunded to all people and the government budget is balanced.

The subsidy affects only the price index part $s_L \frac{\mu}{\sigma-1}\ln\Delta + (1-s_L)\frac{\mu}{\sigma-1}\ln\Delta^*$ in the world welfare function, without influencing the income part due to invariant total world income (perfectly refunded subsidies). The firm relocation to the South increases Δ* and decreases Δ (dΔ =-dΔ*).[25] Since the share of population is smaller in the South ($1-s_L<0.5$), the increased Δ* is dominated by the decreased Δ. Therefore, the subsidy decreases total world welfare.

This welfare-reducing subsidy boils down to a result in Baldwin et al. (2003): The laissez faire outcome in the FC model is Pareto optimal, when $s_K = s_L = s_E$. This implies that a regional subsidy to change firm location is harmful to the world.

Result 11: With an identical capital labour ratio, K/L=K*/L*, one unit of labour and capital owned by each individual in both regions, a subsidy always reduces world welfare. Full agglomeration is socially optimal with zero subsidy, and any anti-agglomeration subsidy should thus be prohibited.

Zero subsidy is socially optimal. In spite of it, it would always be unstable in a decentralised state, in which the periphery would always attempt to introduce a local tax-financed subsidy. Therefore, in order to sustain the zero socially-optimal subsidy, a state should be centralised: the decision making on subsidy policy should be done by a central government with a voting mechanism prohibiting the local tax financing scheme.

5. THE ASYMMETRIC ENDOWMENT MODEL

5.1. *Abundant capital and scarce labour in the core*

The model up to the last section used the same factor endowment ratio, i.e. K/L=K*/L* for simplicity's sake. By contrast, this section models abundant capital but scarce labour endowment in the North (core), i.e. $s_K>0.5$ and $s_L<0.5$ (hence, K/L>K*/L*) keeping the northern relatively large market demand, i.e. $s_E>0.5$ and the typical core-periphery structure, $\phi > \phi^{CP}$.[26] This reflects the current European situation of highly concentrated capital and manufacturing in central Europe but dispersed labour due to sufficient firm mobility but insufficient labour mobility

In this model, each individual supplies <u>one unit of labour</u> and <u>some units of capital</u> (different number of units per-capita across regions). Since the North is abundant in capital endowment but has a small population, a northern individual has more units of capital than one in the South. Note that we keep the assumption of the portfolio in capital returns, in which capital returns <u>per unit of capital</u> are the average of operating profits of all firms and are equal across regions per unit of capital. Thus, capital returns per unit of capital are identical across regions, but capital rewards per capita are different: the

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24 In this section, we do not consider retaliation nor subsidy wars.

25 $\frac{d\Delta}{d\Delta^*} = \frac{\lambda\alpha(\phi-1)a_P{}^{\rho-\sigma}}{\lambda\alpha(1-\phi)a_P{}^{\rho-\sigma}} = -1$

26 We assume relatively and absolutely larger capital endowment in the North and larger labour endowment in the South

northern people own more units of capital than those in the South and capital returns for each northern individual are higher than for southern individuals. The share of expenditure, s_E, is not equal to s_L nor s_K, and can be expressed as

(17) $s_E = \dfrac{s_L(1+\frac{s_K\bar{\pi}}{s_L}-T)}{1+\bar{\pi}-T}$ for global tax-financing scheme and $s_E = \dfrac{s_L(1+\frac{s_K\bar{\pi}}{s_L})}{1+\bar{\pi}-(1-s_L)T^*}$ for local tax-financing scheme.

5.2. *Social welfare and welfare enhancing subsidy*

In the asymmetric endowment model, all features can be kept as in the symmetric endowment ratio model with respect to delocation tendencies and equilibrium. The only major difference in results occurs for social welfare. The welfares per capita are given by

(18) $v = 1+\dfrac{\mu}{\sigma}\dfrac{s_K}{s_L}+\dfrac{\mu}{\sigma-1}\ln(\Delta)$ and $v^* = 1+\dfrac{\mu}{\sigma}\dfrac{1-s_K}{1-s_L}+\dfrac{\mu}{\sigma-1}\ln(\Delta^*)$

Regional welfares and world welfare are given by $V^N = s_L v$, $V^S = (1-s_L)v^*$, and $V^W = s_L v+(1-s_L)v^*$. Figure 11 plots these welfares in terms of subsidy rates. The most interesting contrast is that the world-wide welfare increases in the subsidy. This means that allowing for the subsidy increases world welfare. Also, the global tax-financing scheme is higher than the local tax-financing scheme.

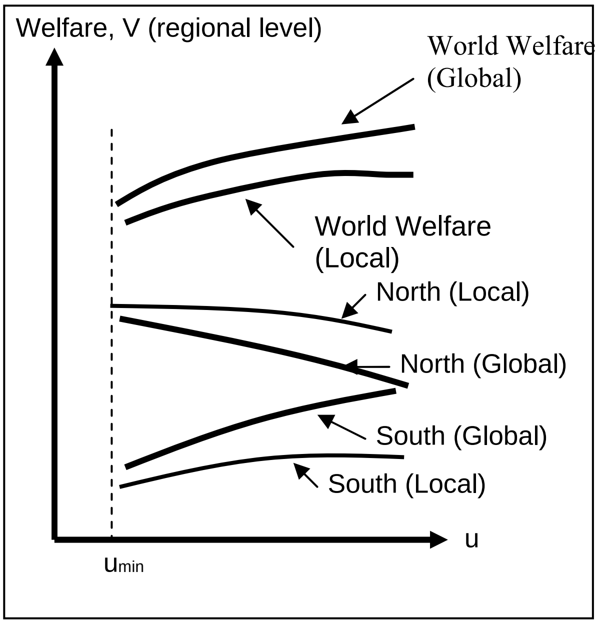

Figure 11: Regional and world welfare and subsidies.

Result 12: If the core region, keeping a larger market size (s_E >0.5), has a larger capital endowment but a smaller labour endowment, i.e. s_L<0.5 and s_K >0.5, then world welfare increases.

Result 13: Global tax-financing scheme is a better policy than local tax-financing scheme as it achieves higher world welfare.

5.3. ***Discussion***

The heart of our discussion is the impact of relocation subsidies on world welfare. In the symmetric endowment ratio model, K/L=K*/L*, a subsidy is harmful, but in the asymmetric factor endowment model, s_K>0.5 s_L<0.5, the subsidy could improve world welfare.[27] Since both models have invariant world total income in V^W, the impact of subsidies is found only in the price index part, i.e. $s_L \frac{\mu}{\sigma-1}\ln\Delta + (1-s_L)\frac{\mu}{\sigma-1}\ln\Delta^*$. First of all, the weight, s_L <0.5 weakens the decreased lnΔ and (1-s_L)>0.5 strengthens the increased lnΔ* and leads to a total increase. Then, the crucial part, lnΔ*, increases in the subsidy. This increase is larger than that of the symmetric endowment ratio model. Starting from the symmetric endowment (s_K=s_L), we fix s_K and decrease s_L such that we keep s_E >0.5. Then, from **(17)** this thought experiment shows a relative decrease of s_E. s_E is lower than in the symmetric model under the same s_K. From **(13)**, we obtain $\frac{\partial^2 a_P}{\partial u \partial s_E} < 0$.[28] What this means is that lowering s_E increases $\frac{\partial a_P}{\partial u}$, so that the subsidy is more responsive and attracts more firms, and increases lnΔ* more than in the symmetric endowment model. For these reasons, world welfare increases in the subsidy.

Next, the global tax financing scheme is favourable for world welfare. As seen in **(15)** and **(16)**, global taxation lowers s_E, compared with southern local taxation. Likewise, $\frac{\partial^2 a_P}{\partial u \partial s_E} < 0$ means that lowering s_E promotes an increase in lnΔ*. Compared with the local tax-financing scheme, the increase of lnΔ* is larger and the decrease in lnΔ is smaller, which increases $s_L \frac{\mu}{\sigma-1}\ln\Delta + (1-s_L)\frac{\mu}{\sigma-1}\ln\Delta^*$ more. Hence the global tax-financed subsidy increases world welfare more.[29] Intuitively, reducing the tax burden in the South by global taxation increases world welfare more, because of the large population in the welfare-increasing South.

Social (world) welfare discussion is crucial in policy analysis. As Baldwin et al (2003) suggested, the stable equilibrium in the FC model is always Pareto optimal for a symmetric endowment ratio. Similarly, in our model, non-subsidy in the same endowment ratio model is optimal. The subsidy decreases world welfare. However, an asymmetric endowment ratio results in an inefficient equilibrium in the non-subsidy case, and there is room for subsidy policy. This subsidy could increase world welfare.

This result is intuitive: allowing for subsidy can enhance welfare not only in the periphery but also in the world. This implies that it could be supportive of today's subsidy policies. In today's EU, factor endowments are more or less asymmetric: while manufacturing is likely to concentrate highly in central Europe, labour is dispersed due to limited labour mobility within Europe (Braunerhjelm et al., 2000; Midelfart-Knarvik and Overman, 2002; Midelfart-Knarvik et al., 2000). If our asymmetric

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[27] Note that s_K>0.5 and s_L<0.5 is a stronger necessary condition than K/L>K*/L*. If s_K> s_L>0.5, its result on world welfare is vague, intermediate between the symmetric and asymmetric endowment ratio models.

[28] See Appendix 3 for derivation.

[29] Note that even if capital income tax is introduced instead of per-capita tax, the results can be qualitatively preserved. It just reduces the tax burden and enhances welfare in the large populated South.

endowment ratio model could be applied, the EU Structural Fund could be rational and meaningful, enhancing social welfare in the whole of the EU.

As long as factor endowments are asymmetric, EU regional subsidy could be welfare-enhancing. However, if labour in the periphery flows into the agglomerated area so as to equalise the endowment ratio, the subsidy would be harmful for social welfare. To make subsidy policy effective, labour migration into agglomerated areas should remain limited or should be prohibited so as to sustain asymmetric endowment ratio. Likewise, if labour migration into agglomerated area is promoted, the subsidy should be prohibited, because the subsidy would decrease social welfare and thus lose its rationality as discussed in section 4.

Another interesting intuition is the way of tax-financing. Global tax financing scheme improves world welfare more than local tax finance does. This implies that an EU committee (central government body) should take initiative for subsidy policy and taxation in all regions equally, rather than local governments. If this model could be applied to the EU's current situation, we can conclude that the global tax-financed subsidy should be initiated by the EU without any national/regional level tax-financed subsidies.

6. Concluding Remarks

This paper studies the impact of relocation subsidies in the HFNEG model in a core-periphery structure. Our main findings are 1) a proportional subsidy induces the most efficient firms to move to the periphery, while a specific subsidy pushes the least efficient firms to the periphery, 2) firm heterogeneity never affects the break and sustain points for a proportional subsidy, whereas it matters for a specific one, 3) non-subsidy is the Nash equilibrium in a centralised state, whereas the periphery in a decentralised state has an incentive to implement local tax-financed subsidies, and there could exist optimal subsidy rates to maximise the welfare in the periphery, and 4) while a subsidy in regions with identical factor endowment ratios deteriorates world welfare, a subsidy in regions with asymmetric factor endowment ratios could enhance world welfare. When endowment ratios are different across regions, global taxation is better than local taxation to finance the subsidy.

Intuitively, since labour is dispersed and industries are concentrated in specific areas in Europe, regional subsidies such as the EU Structural Fund could be rational to enhance social welfare in the core and the periphery keeping limited labour movement toward agglomerated areas. But when allowing for free labour migration which equalises the factor endowment ratios, the rationality for this kind of regional subsidy would go away, and the subsidy should be forbidden.

Possible extensions are many other ways of taxation and different types of subsidy policies, including transfer mechanisms. Also, the political economy of endogenously determined subsidies and other policies with voting mechanism remain for future work. Specifically, voting mechanisms cannot be covered to a further extent by this paper despite its recent importance in the EU, and thus it will need more and deeper analysis in the future.

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APPENDIX 1. LOCAL TAX-FINANCED SPECIFIC SUBSIDY

Here, we prove the hump-shaped southern welfare curve in terms of subsidy rates for local tax-financed specific subsidies to show that there exists at least one maximum level of southern welfare. To show this, noticing that *v** is a monotone and continuous function, it is enough to prove $\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\min}} > 0$ and $\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\max}} < 0$, where $S^{\min}>0$ is to induce the least efficient firms to move and $S^{\max} = +\infty$ is to ensure all firms to move to the south, following Result 1.

$$\frac{\partial v^*}{\partial S} = \frac{\partial \overline{\pi}}{\partial \Delta}\frac{\partial \Delta}{\partial a_S}\frac{\partial a_S}{\partial S} + \frac{\partial \overline{\pi}}{\partial \Delta^*}\frac{\partial \Delta^*}{\partial a_S}\frac{\partial a_S}{\partial S} + \frac{\partial \overline{\pi}}{\partial a_S}\frac{\partial a_S}{\partial S} + \frac{\partial \overline{\pi}}{\partial S} - \frac{\partial T}{\partial a_S}\frac{\partial a_S}{\partial S} - \frac{\partial T}{\partial S} + \frac{\mu}{(\sigma-1)\Delta^*}\frac{\partial \Delta^*}{\partial a_S}\frac{\partial a_S}{\partial S}$$

At $S^{\min}$, ($a_S = 1$) $\frac{\partial \overline{\pi}}{\partial S} = 1 - a_S^{\rho} = 0$, $\frac{\partial T}{\partial S} = \frac{1-a_S^{\rho}}{1-s_E} = 0$, $\frac{\partial a_S}{\partial S} = -\frac{1}{\rho S} < 0$,

$$\frac{\partial \overline{\pi}}{\partial \Delta} = -\frac{\mu}{\sigma\lambda}(1+\phi)s_E < 0, \frac{\partial \overline{\pi}}{\partial \Delta^*} = -\frac{\mu}{\sigma\lambda}\frac{1-s_E}{\phi^2}(1+\phi) < 0, \frac{\partial \overline{\pi}}{\partial \Delta^*} = -\frac{\mu}{\sigma\lambda}\frac{1-s_E}{\phi^2}(1+\phi) < 0$$

$$\frac{\partial T}{\partial a_R} = -\frac{\rho a_S^{\rho} S}{1-s_E} < 0, \frac{\partial \Delta}{\partial a_S} = \lambda(1-\sigma+\rho)(1-\phi), \frac{\partial \Delta^*}{\partial a_S} = \lambda(1-\sigma+\rho)(\phi-1) < 0$$

$$\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\min}} = \left(\frac{\mu}{\sigma}(1+\phi)\left(s_E - \frac{1-s_E}{\phi^2}\right)\right)(1-\sigma+\rho)(1-\phi)\frac{1}{\rho S} + \frac{1}{1-s_E}.$$ Therefore, a sufficient condition for $\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\min}} > 0$ is $s_E - \frac{1-s_E}{\phi^2} > 0$. In other words, if this sufficient condition is satisfied, then $\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\min}} > 0$ always holds.

Next, we show at $\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\max}} < 0$, assuming $a_S = 0$:

$$\frac{\partial \pi}{\partial \Delta} = -\frac{\mu\lambda}{\sigma}\frac{s_E}{\Delta^2}(1+\phi)a_S^{1-\sigma+\rho} = 0, \frac{\partial \pi}{\partial \Delta^*} = -\frac{\mu\lambda}{\sigma}\frac{1-s_E}{\Delta^{*2}}(1+\phi)a_S^{1-\sigma+\rho} = 0,$$

$$\frac{\partial \pi}{\partial S} = 1 - a_S^{\rho} = 1, \frac{\partial T}{\partial S} = \frac{1-a_S^{\rho}}{1-s_E} = \frac{1}{1-s_E}, \quad \frac{\partial T}{\partial a_S} = -\frac{\rho a_S^{\rho} S}{1-s_E} = 0, \frac{\partial \Delta^*}{\partial a_S} = \lambda(\phi-1)(1-\sigma+\rho)a_S^{-\sigma+\rho} = 0$$

Hence, we always get

(19) $$\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\max}} = 1 - \frac{1}{1-s_E} < 0.$$

Therefore, since $\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\min}} > 0$ and $\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\max}} < 0$ always holds for $s_E - \frac{1-s_E}{\phi^2} > 0$, the shape of v^* has at least one hump.

Note that $\left.\frac{\partial v^*}{\partial S}\right|_{S=S^{\max}} = 0$ in global tax-financing scheme, because the denominator in the second term in **(19)** is 1. This corresponds to Result 1 and Figure 7: The maximum level of S is infinite and thus the marginal slope of v^* is 0 at the point.

APPENDIX 2. HUMP-SHAPED WELFARE FOR A PROPORTIONAL SUBSIDY

Following Appendix 1, we prove the hump-shaped curve with respect to u for a southern local tax-financed proportional subsidy. Specifically, since we cannot analytically solve the welfare function in terms of u in an explicit form, we show $\left.\frac{\partial v^*}{\partial u}\right|_{u=u^{\min}} > 0$ and $\left.\frac{\partial v^*}{\partial u}\right|_{u=u^{\max}} < 0$ for local tax financing scheme, which implies at least one hump-shape in the function.

First, we show $\left.\frac{\partial v^*}{\partial u}\right|_{u=u^{\min}} > 0$:

$$\frac{\partial v^*}{\partial u}=\frac{\partial \overline{\pi}}{\partial \Delta}\frac{\partial \Delta}{\partial a_P}\frac{\partial a_P}{\partial u}+\frac{\partial \overline{\pi}}{\partial \Delta^*}\frac{\partial \Delta^*}{\partial a_P}\frac{\partial a_P}{\partial u}+\frac{\partial \overline{\pi}}{\partial a_P}\frac{\partial a_P}{\partial u}+\frac{\partial \overline{\pi}}{\partial u}-\frac{\partial T}{\partial \Delta}\frac{\partial \Delta}{\partial a_P}\frac{\partial a_P}{\partial u}-\frac{\partial T}{\partial \Delta^*}\frac{\partial \Delta^*}{\partial a_P}\frac{\partial a_P}{\partial u}$$
$$-\frac{\partial T}{\partial a_P}\frac{\partial a_P}{\partial u}-\frac{\partial T}{\partial u}+\frac{\mu}{(\sigma-1)\Delta^*}\frac{\partial \Delta^*}{\partial a_P}\frac{\partial a_P}{\partial u}$$

At $u^{\min}$, we assume $a_P=\varepsilon\approx 0$, which is a positive and tiny number, and approximately zero. Thus, for simplification, we assume $a_P > a_P^{\alpha}=\varepsilon^{\alpha}=0$ due to approximation, where $\alpha=1-\sigma+\rho$.

$$\frac{\partial \overline{\pi}}{\partial \Delta}=-\frac{\mu}{\sigma}\lambda\frac{s}{\Delta^2}\left((1-a_P^{\alpha})+a_P^{\alpha}\phi(1+u)\right)=-\frac{\mu}{\sigma}\lambda\frac{s}{\Delta^2},$$
$$\frac{\partial \overline{\pi}}{\partial \Delta^*}=-\frac{\mu}{\sigma}\lambda\frac{1-s}{\Delta^{*2}}\left(\phi(1-a_P^{\alpha})+a_P^{\alpha}(1+u)\right)=-\frac{\mu}{\sigma}\lambda\frac{1-s}{\Delta^{*2}}\phi,\ \ \frac{\partial \Delta^*}{\partial a_P}=\lambda\alpha(1-\phi)a_P^{\rho-\sigma},$$
$$\frac{\partial \Delta}{\partial a_P}=\lambda\alpha(\phi-1)a_P^{\rho-\sigma},\ \ \frac{\partial T}{\partial a_P}=\frac{\mu}{\sigma}(\phi\frac{s_E}{\Delta}+\frac{1-s_E}{\Delta^*})\alpha a_P^{\rho-\sigma}\frac{u}{1-s_E},\ \ \frac{\partial T}{\partial u}=\frac{\mu}{\sigma}(\phi\frac{s_E}{\Delta}+\frac{1-s_E}{\Delta^*})\frac{a_P^{\alpha}}{1-s_E}=0,$$
$$\frac{\partial a_P}{\partial u}=\frac{a_P}{\alpha u}=\frac{\varepsilon}{\alpha u}>0,\ \ \frac{\partial \overline{\pi}}{\partial u}=\frac{\mu}{\sigma}(\phi\frac{s_E}{\Delta}+\frac{1-s_E}{\Delta^*})a_P^{\alpha}=0$$

In sum, $\left.\frac{\partial v^*}{\partial u}\right|_{u=u^{\min}}=\frac{s_E}{\Delta}\left(\left(1+\frac{1}{\phi}\right)+\frac{1}{1-s_E}\left(\frac{1}{\phi}-1\right)\right)\frac{\partial a_P}{\partial u}>0$

At u^{nax} ($a_P=1$), we show $\left.\frac{\partial v^*}{\partial u}\right|_{u=u^{\max}}<0$:

$$\frac{\partial \overline{\pi}}{\partial \Delta}=-\frac{\mu}{\sigma}\lambda\frac{s}{\Delta^2}\left((1-a_P^{\alpha})+a_P^{\alpha}\phi(1+u)\right)=-\frac{\mu}{\sigma}\lambda\frac{s}{\Delta^2}\phi(1+u),$$
$$\frac{\partial \overline{\pi}}{\partial \Delta^*}=-\frac{\mu}{\sigma}\lambda\frac{1-s}{\Delta^{*2}}\left(\phi(1-a_P^{\alpha})+a_P^{\alpha}(1+u)\right)=-\frac{\mu}{\sigma}\lambda\frac{1-s}{\Delta^{*2}}(1+u)\ \ \frac{\partial \Delta^*}{\partial a_P}=\lambda\alpha(1-\phi)a_P^{\rho-\sigma}=\lambda\alpha(1-\phi),$$
$$\frac{\partial \Delta}{\partial a_R}=\lambda\alpha(\phi-1)a_P^{\rho-\sigma}=\lambda\alpha(\phi-1),\ \frac{\partial T}{\partial a_R}=\frac{\mu}{\sigma}(\phi\frac{s_E}{\Delta}+\frac{1-s_E}{\Delta^*})\alpha a_P^{\rho-\sigma}\frac{u}{1-s_E}=0,$$
$$\frac{\partial T}{\partial u}=\frac{\mu}{\sigma}(\phi\frac{s_E}{\Delta}+\frac{1-s_E}{\Delta^*})\frac{a_P^{\alpha}}{1-s_E}=0,\ \frac{\partial a_P}{\partial u}=\frac{a_P}{\alpha u}=\frac{1}{\alpha u^{\max}},\ \ \frac{\partial \overline{\pi}}{\partial u}=0,$$

Using these solutions, the derivative is

$$\left.\frac{\partial v^*}{\partial u}\right|_{u=u^{\max}}=\left((1-\phi)\left((1+u^{\max}-\frac{u^{\max}}{1-s_E})(\frac{s_E}{\phi}-(1-s_E))+\frac{\sigma}{\sigma-1}\right)-\frac{u^{\max}}{1-s_E}\right)\frac{1}{u^{\max}}-\frac{s_E}{1-s_E}$$

As shown in Figure 12, we always get $\left.\frac{\partial v^*}{\partial u}\right|_{u=u^{\max}}<0$ for any values of parameters under $\phi>\phi^{CP}=\frac{1-s_E}{s_E}$.[30] Hence, southern welfare curve in terms of *u* is always hump-shaped.

[30] The condition $\phi>\phi^{CP}$ cuts the sphere in Figure 12 as a triangle shape in the 3-dimension of to s and ϕ—small s>0.5 and small ϕ cannot satisfy this condition.

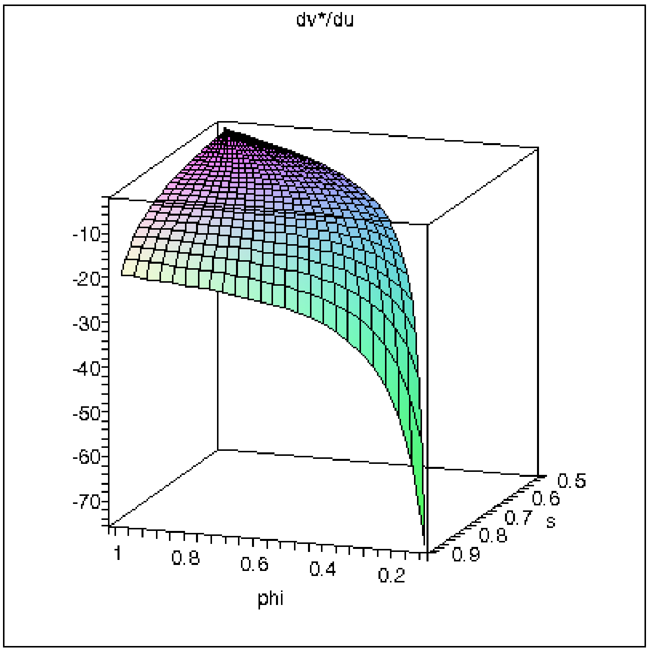

Figure 12: $\frac{\partial v^*}{\partial u} < 0$ **in terms of ϕ and sE (σ=2).**

APPENDIX 3 THE CUT-OFF LEVEL AND MARKET SIZE

To ensure $\frac{\partial^2 a_P}{\partial u \partial s_E} < 0$, $\frac{\partial a_P}{\partial s_E} < 0$ is derived in this Appendix ($\frac{\partial a_P}{\partial u} > 0$ was already derived in Appendix 2). Using

(13), $\frac{\partial a_P}{\partial s_E} = \frac{\partial a_P}{\partial \chi}\frac{\partial \chi}{\partial s_E} = -\frac{1}{\alpha}\left(\frac{1-\phi\chi}{(1-\phi)(1+\chi)}\right)^{1/\alpha - 1}\frac{1+\phi}{(1-\phi)(1+\chi)^2}\left(\frac{(1-\phi(1+u))}{(1+u-\phi)}\right)\frac{1}{(1-s_E)^2} < 0$

where $a_P = \left(\frac{1-\phi\chi}{(1-\phi)(1+\chi)}\right)^{1/\alpha} > 0$ and $\chi \equiv \frac{(1-\phi(1+u))s_E}{(1+u-\phi)(1-s_E)}$. Note that χ>0 because (1-ϕ(1+u))>0 can be derived from maximum level of u, shown in Result 2: $1+u < 1+u^{\max} = \frac{s_E}{\phi} + \phi(1-s_E) < \frac{1}{\phi}$.