THE GRADUATE INSTITUTE | GENEVA

INSTITUT DE HAUTES ETUDES
INTERNATIONALES ET DU DÉVELOPPEMENT

GRADUATE INSTITUTE OF INTERNATIONAL
AND DEVELOPMENT STUDIES

Graduate Institute of
International and Development Studies Working Paper
No: 03/2012

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February 16, 2012

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This paper discusses fiscal policy using a DSGE model with search and matching in the labour market. Fiscal policy is effective mainly via its impact through the labour market. Although public intervention tends to crowd out private consumption, public spending also improves the matching between unemployed workers and job vacancies. The mechanism modeled in this paper shares similarities with Baxter and King (1993) and Leeper *et al.* (2010). The model produces positive fiscal multipliers on impact and in the short term and consistently reproduces the reaction to a spending shock of the main labour market variables such as wages, employment or labour market tightness. These results are similar with that of Monacelli *et al.* (2010) except that the transmission channel does not depend on the downward adjustment of the reservation wage of workers. The size of the fiscal multiplier increases with the elasticity of matching to spending and is also negatively related with the steady state spending to GDP ratio in the presence of diminishing marginal returns on spending. For large value of the multiplier, there is a crowding in of consumption and investment. Lastly, this model produces output multipliers larger than 1 in the presence of nominal price rigidities.

Keywords: Fiscal policy, search, matching.

JEL CLASSIFICATION SYSTEM: E24, E32, E62.

*The authors wish to thank Ekkehard Ernst and Stefan Kuhn for helpful comments as well as the participants of the research seminar in international economics of the Graduate Institute in Geneva. The usual disclaimers apply.

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1 Introduction

The recent episodes of fiscal stimulus and fiscal consolidation raise the issue of the efficiency of fiscal policy. Empirical estimations of the fiscal multiplier vary widely ranging from 0 to above 1.[1] A consensus exist on the positive reaction of consumption to spending shocks as in Blanchard and Perotti (2002) to the exception of Hall (2009), while results on investment are inconclusive. Mountford and Uhlig (2009) finds a positive reaction of investment in contrast with Ramey (2011). Only few studies discuss the reaction of the main labour market variable to a spending shock. Monacelli and Perotti (2008) points to an increase in hours and real wages. Yuan and Li (2000) underline the inverse reaction of hours and employment in a model with a matching mechanism. Contrastingly, Monacelli *et al.* (2010) argues that most of the adjustment takes place along the extensive margin and that both the probability to find a job and labour market tightness increase following fiscal expansion.

This paper develops a model of fiscal policy, which produces positive output and employment multipliers. The model also accounts for the positive reaction of wages, job finding probability and labour market tightness to a spending shock. The model developed is a RBC model with a public sector and search and matching function. An innovation is that the traditional matching function is extended to incorporate public spending. The Cobb-Douglas matching function is now made of three elements: searching workers, vacancies and public spending. There are ambivalent transmission channels. Fiscal spending crowds out private consumption and investment in line with the Ricardian properties associated with intertemporal optimizing private agents. The increase in the interest rate following fiscal expansion also affects the discounted value of an additional match and reduces the incentive of firms to hire an additional worker. Labour market spending however improves the functioning of the labour market and increases the rate of matching. In this model, public spending create a positive externality on the labour market similarly to Baxter and King (1993) and Leeper *et al.* (2010), in which public spending has a positive externality on firms production. A more restrictive interpretation is that public spending improves the efficiency of the labour market when it takes the form of active labour

[1] There are three main methodologies to estimate the size of the fiscal multiplier empirically: the dummy approach Ramey and Shapiro (1998), structural VAR such as Blanchard and Perotti (2002), Monacelli and Perotti (2008) or Galí *et al.* (2007) or sign restriction Mountford and Uhlig (2009). Christiano *et al.* (2009) points that the multiplier is substantially larger than 1 when monetary policy reaches the zero lower bound.

market spending.

Various forms of matching functions are discussed. The benchmark matching function has constant return to scale on the three inputs. Alternatives specifications consider the case where the matching function has constant return to scale on searching workers and vacancies with government spending being nested with one of the two other inputs. We also discuss the impact of the steady state value of public spending on the size of the multiplier and on the outcome of the main macroeconomic variables. Diminishing marginal return on spending implies that low steady state spending to GDP ratio generates large fiscal multiplier and a crowding in of consumption and investment. Lastly, the case of a CES matching function is considered to better account for the dynamic of vacancies following a spending shock.

There is growing related literature. Monacelli *et al.* (2010) (MPT thereafter) also focus on the transmission of fiscal spending through the labour market. They use a utility function *à la* Shimer (2005) where the negative wealth effect associated with higher public spending enters negatively the surplus from an additional match. Workers lower their reservation wage, which increases the incentive for firms to hire. We use instead the search and matching framework developed by Ravenna and Walsh (2008) in which the reservation wage of workers depends on unemployment benefits. We are therefore able to isolate the contribution of spending in improving employment.

Related papers also include Ganelli (2003), which introduces non-separability between private and public consumption to discuss the size of the fiscal multiplier in an open economy framework. Baxter and King (1993) consider the case of private firms using public investment as an input for production. Leeper *et al.* (2010) uses a similar mechanism and estimate the model for the U.S.. Galí *et al.* (2007) use non-optimizing households to break the Ricardian equivalence. The GIMF model also rely on rule of thumb households as well as overlapping generation to assess the size of the multiplier in an multizone model Freedman *et al.* (2010). Monacelli and Perotti (2008) combine a GHH utility function with sticky prices to produce positive fiscal multipliers, while Ravn *et al.* (2006) make use of deep habit formation and sticky price for a similar purpose. Lastly, Fernández-Villaverde (2010) shows that fiscal multipliers are positive when public consumption takes place in the presence of financial frictions, while Challe and Ragot (2011) argue that debt financed spending reduces the liquidity constraint of firms.

The paper is organized as follow. Section 3 presents the model and the main assumptions. Section 4 discusses the steady states and the calibration, while section 5 details the properties of the model by use of numerical simulations. Section 6 concludes.

2 The model

2.1 Unemployment, Vacancies and Matching

At the beginning of each period, the workforce L, which is normalized to 1, is divided between employed workers n_t and unemployed workers $u_t = 1 - n_t$. The number of workers currently employed at time t is equal to the existing stock of employment at the beginning of the period ρn_{t-1} plus new matches m_t. The rate of job survival is a constant ρ:

$$n_t = \rho n_{t-1} + m_t \tag{1}$$

New matches m_t depend positively on the number of searching workers $s_t = 1 - \rho n_{t-1}$ and the number of vacancies v_t. The innovative feature of this model is to introduce labour market spending g_t into the matching function. The main motivation is that spending aim at improving the matching between searching workers and vacancies. The matching function is similar to a Cobb-Douglas production function, with σ_i $(i = s, v, g)$ the elasticities of substitution between the different inputs. The parameter σ_m reflects the efficiency of the matching process. The value of the different elasticities of substitution matters for the dynamic of the economy, as they influence the return to scale of the matching function. It is assumed through out the paper that the matching function has constant return to scale. The sum of the elasticities of substitution is equal to one $\sigma_v = 1 - \sigma_s - \sigma_g$.

$$m_t = \sigma_m s_t^{\sigma_s} v_t^{\sigma_v} g_t^{\sigma_g} \tag{2}$$

Two alternative specifications of the matching function will be considered against this benchmark case further below. First, labour market spending may be associated with vacancies: $m_t = \sigma_m s_t^{\sigma_s} \left(g_t^{\sigma_g} v_t \right)^{\sigma_v}$ with $\sigma_v = 1 - \sigma_s$. Second, labour market spending may be associated with unemployment: $m_t = \sigma_m \left(g_t^{\sigma_g} s_t \right)^{\sigma_s} v_t^{\sigma_v}$ with $\sigma_v = 1 - \sigma_s$.

For convenience, we use the ratio $\theta_{s,t} = \frac{v_t}{s_t}$ to measure labour market tightness. The ratio $\theta_{g,t} = \frac{g_t}{v_t}$ measures labour market spending per vacancy. The probability of firms to fill up a vacancies is denoted $q(\theta_{s,t}; \theta_{g,t})$ and is equal to the ratio of matches over the number of vacancies:

$$q_t = \frac{m_t}{v_t} = \sigma_m s_t^{\sigma_s} v_t^{\sigma_v - 1} g_t^{\sigma_g} = \sigma_m \theta_{s,t}^{-\sigma_s} \theta_{g,t}^{\sigma_g} \tag{3}$$

Similarly, the probability of an unemployed workers to find a job is given by the ratio of new matches over the number of searching workers.

$$p_t = \frac{m_t}{s_t} = \sigma_m s_t^{\sigma_s - 1} v_t^{\sigma_v} g_t^{\sigma_g} = \sigma_m \theta_{s,t}^{1-\sigma_s} \theta_{g,t}^{\sigma_g} \tag{4}$$

Both probabilities of firms to fill up a vacancies and of searching workers to find a job are now increasing with public spending. Given the above definitions, new matches in the equation for n_t can be expressed as the probability of filling a vacancy times the existing number of vacancies:

$$n_t = \rho n_{t-1} + q_t v_t \tag{5}$$

2.2 Households:

Households maximize their expected intertemporal utility function $U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ by choosing the optimal level of consumption c_t as well as the level of investment of firms x_t and the quantity of public bonds held b_t. Employed workers receive real wage w_t and unemployed households receive a replacement income w^u, which is a fraction of the steady state value of wages. A large representative household with a continuum of members of mass one inhabits the artificial economy. All household's members pool their incomes to be fully insured against unemployment. The intertemporal budget constraint of a household member is given by:

$$c_t + x_t + b_t \leq w_t n_t + w^u (1 - n_t) + r_{k,t} k_{t-1} + r_{t-1} b_{t-1} - \tau_t + \Pi_t \tag{6}$$

where $r_{k,t}$ denotes the real rental rate of capital k_t, r_t the interest on public bonds, τ_t is a lump-sum tax, Π_t is the profits received from firms. The representative household also faces an employment constraint in the labor market, with m_t being expressed as a function of the probability of unemployed members to find a job and the fraction of searching members.

$$n_t = \rho n_{t-1} + p_t s_t \tag{7}$$

Capital accumulation is subject to the following constraint:

$$k_t = (1 - \delta) k_{t-1} + x_t (1 - \phi_t) \tag{8}$$

where δ is a parameter for the rate of capital depreciation, $\phi_t \equiv \frac{\phi}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2$ denotes the capital adjustment costs that are proportional to the rate of change in investment, with $\phi(0) = \phi'(0) = 0$. The agent's optimization problems stated above focus only on interior solutions, that is all the quantities are supposed to be strictly positive. The representative household's optimization problem can be restated with a bellman equation as follows:

$$H(n_{t-1}, k_{t-1}, b_{t-1}, x_{t-1}) = \max_{c_t, n_t, x_t, k_t, b_t} \left(\frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_t \{ H(n_t, k_t, b_t, x_t) \} \right) \tag{9}$$

subject to(6), (7) and (8). The first order condition for consumption links the marginal utility of wealth with the marginal utility of consumption:

$$\lambda_t = \frac{1}{c_t^{\sigma}} \tag{10}$$

where λ_t is Lagrange multiplier associated with the budget constraint (6). The first order conditions for investment and capital read:

$$\varphi_t \left[1 - \left(\phi_t + \frac{x_t}{x_{t-1}} \phi_t^{'}\right)\right] = 1 - \beta E_t \left\{\varphi_{t+1} \Lambda_{t,t+1} \left(\frac{x_{t+1}}{x_t}\right)^2 \phi_{t+1}^{'}\right\} \tag{11}$$

$$\varphi_t = \beta E_t \left(\Lambda_{t,t+1} \left[r_{k,t+1} + \varphi_{t+1} (1 - \delta)\right]\right) \tag{12}$$

with $\Lambda_{t,t+1}\text{is defined as } \frac{\lambda_{t+1}}{\lambda_t}$. φ_t is the shadow value of a unit of investment and $\phi_t^{'}$ the derivative of the capital cost function with respect to its argument $\frac{x_t}{x_{t-1}} - 1$. The first order condition with respect to public bonds can be expressed as follow:

$$\frac{1}{r_t} = \beta_t E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\right\} \tag{13}$$

We derive $H_{n,t}$ the representative household's marginal value of having one of its member hired in the labor market rather than unemployed, which enters further below the Nash wage bargaining. $H_{n,t}$ increases with additional income gains expressed in utility from being employed rather than unemployed. $H_{n,t}$ also increases with the expected utility of being still employed in the next period.

$$H_{n,t} = \lambda_t (w_t - w^u) + \beta \rho E_t \left\{H_{n,t+1} (1 - p_{t+1})\right\} \tag{14}$$

2.3 Firms

The economy is populated by a large number of identical firms of mass one, which operate in a competitive goods market. In order to produce the final goods, y_t, firms use a constant return to scale Cobb-Douglas technology with two inputs labor n_t and capital k_{t-1} of the form:

$$y_t = f(k_{t-1}, n_t) = (k_{t-1})^{\zeta} (n_t)^{1-\zeta} \tag{15}$$

The representative firm posts v_t vacancies in the beginning of the period in order to increase its quantity of labor input, n_t. It is costly for the representative firm to post a vacancy. Specifically, the vacancy cost function, $C(v_t)$, is assumed to be linear in the number of posted vacancies: $C(v_t) = \kappa v_t$, where $\kappa > 0$ is a vacancy cost parameter. m_t new matches come out of the searching mechanism. Then, firms use the new matches — as well as of the other inputs — to produce a homogenous good y_t. Formally, from the firms side, the employment evolves over time following equation 5: $n_t = \rho n_{t-1} + q_t v_t$. Firms maximize the expected flows of profits $E_t \sum_{j=0}^{+\infty} \beta^j \Lambda_{t,t+j} \Pi_{t+j}$ with respect to employment n_t and capital k_{t-1} and subject to the production function and the law of motion of employment equations 15 and 5. Profit,Π_t is made of the value of output minus labour costs, the cost of renting capital and the costs of vacancies:

$$\Pi_t \equiv y_t - w_t n_t - r_{k,t} k_{t-1} - \kappa v_t \tag{16}$$

The representative firm's optimization problem can be restated with a bellman equation:

$$F(n_{t-1}, k_{t-1}) = \max_{n_t, k_{t-1}} \left((k_{t-1})^{\zeta} (n_t)^{1-\zeta} - w_t n_t - r_{k,t} k_{t-1} - \kappa v_t + \beta E_t \left\{ \Lambda_{t,t+1} F(n_t, k_t) \right\} \right) \tag{17}$$

subject to (5). The first order conditions of the firm's optimization problem with respect to k_{t-1} equates the productivity marginal of capital with the rental rate of capital:

$$r_{k,t} = \zeta \frac{y_t}{k_{t-1}} \tag{18}$$

Firms first choose the optimal quantity of vacancies $-\psi_t = \frac{\kappa}{q_t}$. Then maximizing profits with respect to employment and making use of the envelope condition, we get the equilibrium condition for employment:

$$\frac{\kappa}{q_t} = a_t - w_t + \beta \rho E_t \left\{ \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \right\} \tag{19}$$

where $a_t \equiv (1-\zeta) \frac{y_t}{n_t}$ is the marginal productivity of labour. In equilibrium, the marginal cost of hiring a workers is equal to its marginal benefits. The latter is the difference between the marginal productivity of an additional jobs and its wage costs at time t, plus savings from not having to hire an additional workers at time $t+1$.

It is necessary to define the value for a firms of an additional workers $F_{n,t}$, which enters the wage bargaining in the following section. Using the employment condition for employment and making use of the equilibrium condition for posting vacancies, we get that $F_{n,t} = \frac{\kappa}{q_t}$. Plugging this definition into equation 19 yields:

$$F_{n,t} = a_t - w_t + \beta \rho E_t \left\{ \Lambda_{t,t+1} F_{n,t+1} \right\} \tag{20}$$

2.3.1 Nash bargaining in the labor market and surplus

Each period, the real wage in the formal labor market is determined through a generalized Nash-bargaining process between the representative firm and the marginal worker that was matched with the firm. Formally,

$$w_t \equiv max \left\{ (H_{n,t})^{\eta} (F_{n,t})^{1-\eta} \right\}, \quad 0 < \eta < 1 \tag{21}$$

where η denotes the bargaining power of the workers and where the expressions of $H_{n,t}$ and $F_{n,t}$ are given by (14) and (20), respectively. The first order condition of the Nash-bargaining process is given by

$$\eta F_{n,t} = (1-\eta) \frac{H_{n,t}}{\lambda_t} \tag{22}$$

where $\frac{H_{n,t}}{\lambda_t}$ represents the household's marginal value of an additional worker expressed in units of consumption goods. The total surplus from a marginal match in the labor market (or surplus for short), denoted by $S_{n,t}$, is defined as the sum of the firm's marginal value of an additional hiring a worker and the household's marginal value of an additional worker defined in units of consumption goods: $S_{n,t} \equiv F_{n,t} + \frac{H_{n,t}}{\lambda_t}$. Straightforwardly, one can show that the Nash-bargaining process leads the household and the firm to share that surplus: $F_{n,t} = (1-\eta) S_{n,t}$ and $\frac{H_{n,t}}{\lambda_t} = \eta S_{n,t}$. In addition, the surplus $S_{n,t}$ can also be measured by the size of the gap between firm's reservation wage $\overline{w}_t$ and the household's reservation wage $\underline{w_t}$:

$$S_{n,t} = \bar{w}_t - \underline{w}_t \tag{23}$$

The household's reservation wage $\underline{w_t}$ defines the minimum value of the real wage for which the household is willing to work in the labour market. In turn, firm's reservation wage $\overline{w}_t$ defines the maximum value of the real wage that firms are willing to pay a worker. The household's marginal value of an additional worker expressed in units of consumption goods becomes zero $\frac{H_{n,t}}{\lambda_t} = 0$ if the real wage is set equal to the household's reservation wage $w_t = \underline{w}_t$. In this case, equation (14) becomes:

$$\underline{w}_t \quad = \quad w^u - \beta \rho E_t \left\{ \Lambda_{t,t+1} \frac{H_{n,t+1}}{\lambda_{t+1}} (1 - p_{t+1}) \right\} \tag{24}$$

The household's reservation wage, $\underline{w}_t$, increases with the replacement wage, w^u. In turn, $\underline{w}_t$ decreases with the household's expected future continuation value of the match, $\beta E_t \Lambda_{t,t+1} \frac{H_{n,t+1}}{\lambda_{t+1}} \rho (1 - p_{t+1})$. Similarly, the firm's marginal value of an additional hiring of a worker is zero $F_{n,t} = 0$, if the real wage is set equal to the firm's reservation wage, $w_t = \bar{w}_t$. In this case, equation (20) becomes:

$$\bar{w}_t \quad = \quad a_t + \beta \rho E_t \left\{ \Lambda_{t,t+1} F_{n,t+1} \right\} \tag{25}$$

Firm's reservation wage, $\bar{w}_t$, increases with the current marginal productivity of labor and with the firm's expected future continuation value of the match. This last element reflects that turn over is costly for firms. The bargained real wage, w_t, is then obtained by taking the average sum of the two reservation wages, the weights being given by the bargaining powers of firms and households:

$$w_t = \eta \bar{w}_t + (1 - \eta) \underline{w}_t \tag{26}$$

Equation 26 can be rearrange by using equations 24, 25 together with $F_{n,t} = \frac{\kappa}{q_t}$ and $\frac{H_{n,t}}{\lambda_t} = \frac{\eta}{1-\eta}\frac{\kappa}{q_t}$:

$$w_t = \eta a_t + (1-\eta) w^u + \eta \beta \rho \kappa E_t \left\{ \Lambda_{t,t+1} \frac{p_{t+1}}{q_{t+1}} \right\} \tag{27}$$

The real wage is a weighted sum of the marginal productivity of labour and the replacement income at time t and the the expected future state of the labour market at time $t+1$. The weights are made of the bargaining power of firms and workers. We can also compute a recursive expression for the surplus, $S_{n,t}$, by plugging (24) and (25) into (23) and by using the relations between the surplus, $S_{n,t}$, and the marginal values of an additional labor, $H_{n,t}$ and $F_{n,t}$:

$$S_{n,t} = (a_t - w^u) + \beta \rho E_t \left\{ \Lambda_{t,t+1} S_{n,t+1} (1 - \eta p_{t+1}) \right\} \tag{28}$$

The surplus that arises from the current match is determined by two terms (appearing in the right-hand side of equation (28)). $S_{n,t}$ increases with the gap between the marginal productivity of labor and the replacement wage w^u. The current surplus also increases with the expected next period surplus, if the current match is not broken in the following period, $\beta \rho E_t \left\{ \Lambda_{t,t+1} S_{n,t+1} \right\}$, net of the expected next period household's marginal value of an additional worker (expressed in units of consumption goods), derived from a new match that would occur in the following period, $\beta \eta \rho E_t \left\{ \Lambda_{t,t+1} p_{t+1} S_{n,t+1} \right\} = \beta \rho E_t \left\{ \Lambda_{t,t+1} p_{t+1} \frac{H_{n,t+1}}{\lambda_{t+1}} \right\}$.

Recall that a fraction $1-\eta$ of the surplus goes to firms: $F_{n,t} = (1-\eta) S_{n,t}$ and that $F_{n,t} = \frac{\kappa}{q_t}$, we get a recursive equation reflecting the dynamic of employment:

$$\frac{\kappa}{q_t} = (1-\eta)(a_t - w^u) + \beta \rho E_t \left\{ \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} (1 - \eta p_{t+1}) \right\} \tag{29}$$

When either the vacancy posting cost parameter becomes close to zero,$\kappa \to 0$, or the matching efficiency parameter strongly improves, $\alpha_m \to +\infty$, the marginal productivity of labor is equal to the replacement wage in the equilibrium.

This expression slightly differs from the corresponding equation in MPT to the extent that the marginal value of non-work activities is replaced by w^u. This difference comes from the choice of utility function. In this model, employment does not enter the utility function negatively, while MPT uses a utility function similar to that of Shimer (2005). In MPT, the lower value of non-work activities following a spending shock reduces the reservation wage of workers. The incentive for firm to hire increases and lead to a positive fiscal multiplier. The absence of this transmission channel between fiscal policy and employment in this model enables to control that public spending affects employment directly through the new formulation of the matching function.

2.3.2 Government policy and resource constraint:

The government issue bonds b_t to finance the difference between tax income and spending. Public debt increases with the last period stock of debt and the associated interest payments $r_{t-1}b_{t-1}$and with the primary deficit d_t. The primary deficit is the difference between tax income and labour market spending. Taxes are lump sum in this simple version of the model. Public spending are made of g_t as well as unemployment benefits $w^u(1-n_t)$.

$$b_t = r_{t-1}b_{t-1} + d_t \quad (30)$$

$$d_t = g_t + w^u(1-n_t) - \tau_t \quad (31)$$

Public consumption g_t follows an auto-regressive process. Taxes τ_t also follow an auto-regressive process and adjust as well to the level of public debt.

$$g_t = (1-\rho_g)g + \rho_g g_{t-1} + \varepsilon_{I,t} \quad (32)$$

$$\tau_t = (1-\rho_\tau)\tau + \rho_\tau \tau_{t-1} + \tau_b(b_{t-1} - b) \quad (33)$$

The resource constraint states that aggregate demand equals the sum of private consumption, investment, search costs and labour market spending. In the absence of nominal price stickiness, y_t is determined by the supply side. The resource constraint implies that labour market spending crowds out private consumption and investment.

$$y_t \quad = \quad c_t + x_t + \kappa v_t + g_t \tag{34}$$

3 Steady states and calibration

The equations of the model are presented in appendix B. The model contains 16 parameters, gathered in the set

$$\Theta \equiv \left\{\beta,\, \delta,\, \zeta,\, \eta_k,\, \rho_g,\, \rho,\, \sigma_g\, \sigma_s,\, \eta,\, \sigma,\, \sigma_m,\, \kappa,\, \alpha_u,\, \rho_\tau,\, \rho_b,\, w^u\right\}$$

and 20 variables, which steady state levels are gathered in the set

$$\Delta \equiv \left\{y,\, c,\, x,\, n,\, k,\, w,\, r_k,\, v,\, p,\, q,\, \theta_s\,,\, \theta_g\,,\, a,\, \lambda,\, \varphi,\, g,\, b,\, r,\, d,\, \tau_t\right\}$$

In order to assign a value to each parameter and steady state variable of the model, we solve a system formed by the deterministic steady-state expressions of 19 equilibrium equations (20 equations minus auto-correlation of the public spending shock) and 17 additional restrictions. The 17 additional restrictions are put on a subset of steady state variable levels, $\underline{\Delta} \equiv \left\{p,\, \theta_s,\, \theta_g\right\}$, on a subset of the model parameters $\underline{\Theta} \equiv \left\{\beta,\, \delta,\, \zeta,\, \eta_k,\, \rho_g,\, \rho,\, \sigma_g\, \sigma_s,\, \eta,\, \sigma,\, \alpha_u,\, \rho_\tau,\, \rho_b\right\}$, and finally on a steady state ratio, $\Gamma \equiv \left\{\frac{b}{y}\right\}$. We assume that the time unit is a semester, similarly to Ravenna and Walsh (2008). The calibration of $\underline{\Delta}$, $\underline{\Theta}$, and Γ which is based on postwar U.S. data, is discussed below.

The steady state can be found in appendix C. From eq 11, the shadow value of a unit of investment φ is 1. Both interest rates on bond and capital are the inverse of the discount factor (eq 13), the interest rate on capital being also adjusted for depreciation (eq 12). The probability to find a job p and labour market tightness θ_s are set at 0.45 and 0.5 respectively following Shimer (2005). Spending per vacancy is set at 0.45 to produce a realistic ratio of spending to GDP (see below). It follows that the efficiency of the matching process σ_m is given by eq 4. The steady states for the probability of firms to fill a vacancy q, vacancy v and employment n are given by eq 3, by the definition of θ_s and by eq 5.

Using eq 18 and 15, we get the steady state values for the stock of capital and output expressed as a function of the output to capital ratio. Investment grows with depreciation following eq 8, while the marginal productivity of labour is a simple definition $a \equiv (1-\zeta)\frac{y}{n}$. we get the steady state values of w^u and κ by solving equations 27 and 29. Wages are linked to the steady state unemployment benefit by the inverse of the replacement rate α^u. Public spending is given by the definition of θ_g, while consumption is the residual from the resource constraint (eq 34). Lastly, the steady state for public bonds, tax and deficit are given by eq 30 and 31 knowing that the steady state debt to output ratio is fixed at 60%.

The calibration follows Ravenna and Walsh (2008) for the parameters of the labour market. The calibration of the discount factor is set so that the annual real interest rate is approximately 4 percent on average. The capital is assumed to depreciate at the rate of 10 percent on an annual basis. Around one third of the economy's income goes to capital owners. Specifically, the discount factor, the capital depreciation rate and the capital share in the production function are set to $\beta = 0.99$, $\delta = 0.025$, and $\zeta = \frac{1}{3}$. These values are standard in the business cycle literature. The parameter for the adjustment of capital cost is set to $\eta_k = 0$ such that we ignore these costs initially. The survival rate in the labor market is set to $\rho = 1 - 0.1$. The elasticity of matches to searching workers parameter is set to $\sigma_s = 0.6$, in line with existing estimations ranging between 0.5 and 0.7 in the literature. The bargaining power between firms and households is $\eta = 0.6$ to meet the Hosios condition for efficiency. We set $p = 0.45$ to meet the estimates of the job finding rates and $\theta_s = 0.5$ to meet the measures of US vacancies Shimer (2005). We calibrate θ_g to 0.45 such that the spending to GDP ratio corresponds to 1.2% the average labour market spending to GDP ratio in OECD countries. [2] As a benchmark, Leeper *et al.* (2010) set the ratio of public investment to GDP at 3.8%. The elasticity of matches to public employment σ_g is set at 0.05. This parameter is crucial for the size of the multiplier, although there is no estimate of its value to our knowledge. Sensitivity analysis with respect to this parameter is performed in section 4. Considering the elasticity of production to public capital, both Baxter and King (1993) and Leeper *et al.* (2010) use a similar value. The debt to GDP ratio is also fixed at 60% and the auto-regressive parameter of the government spending and taxes stochastic process are set

[2] We use the active labour market spending data from the social expenditure database published by the OECD.

Table 1: Parameters and main steady state values

Capital depreciation rate in the production function	δ	0.025
Discount factor	β	0.99
Capital share	ζ	$1/3$
Capital adjustment cost parameter	η_k	0
Government auto regressive parameters	ρ_g, ρ_τ	0.9
Survival rate	ρ	0.9
Elasticity of matches to unemployment	σ_s	0.6
Elasticity of matches to public spending	σ_g	0.05
Bargaining power	η	0.6
Probability of finding a job	p	0.45
Labor market tightness	θ_s	0.5
Spending per vacancy	θ_g	0.45
Government spending share in output	$\frac{g}{y}$	1.2%
Debt to GDP ratio	$\frac{b}{y}$	60%
Consumption to GDP ratio	$\frac{c}{y}$	70%
Unemployment rate	u	10%

to $\rho_g = \rho_\tau = 0.9$. Table 1 below summarized the restrictions on the model parameters and on the model steady states.

4 Results

This section rely on numerical simulations to investigate the properties of the model. We study in particular the dynamic of the economy following a positive shock on public spending and for different configurations of the economy. The impulse response functions are displayed in percentage deviation from steady state following a spending shock equal to 1 percent of GDP. Employment as well as labour market tightness and job finding probabilities are in absolute term.

4.1 Baseline results

Figure 1 displays the dynamic of the economy following a positive shock on public spending (solid line). The effects of spending are a mix between the resource effect associated with higher spending and a productivity shock on matching efficiency. For the sake of comparison, the case corresponding to an increase in government spending (without matching effect, $\sigma_g = 0$, dashed line) and the case corresponding to a matching efficiency shock (no spending shock, dot line) are also represented in figure 1. The share of spending in output is 1.2 percent at the steady state in line with the average level of active labour market spending in OECD countries. The spending shock amounts to 1 percent of GDP leading spending to increase to 2.2 percent of GDP at the time of the shock.

The impact of public spending on employment appears in eq 29, which can be rearranged as follow:

$$\frac{\kappa}{\sigma_m}\theta_{s,t}^{\sigma_s} = \theta_{g,t}^{\sigma_g}\left[(1-\eta)(a_t - w^u) + \rho\frac{\kappa}{\sigma_m r_t}E_t\left\{(1-\eta p_{t+1})\frac{\theta_{s,t+1}^{\sigma_s}}{\theta_{g,t+1}^{\sigma_g}}\right\}\right] \tag{35}$$

Labour market tightness $\theta_{s,t}$ is decreasing with the interest rate r_t. Following a shock on public spending, the rise in the interest rate lowers the discounted benefit from an additional match. Firms surplus from an additional match drops, affecting negatively vacancy posting. The interest rate also has an indirect effect on hiring decisions through its impact on capital accumulation. Higher interest rate discourages investment and leads to a decline in the marginal product of labour. Lower surplus from an additional match further discourages vacancy posting. These two negative effects are counter-balanced by the positive effect of spending per vacancy $\theta_{g,t}$ on labour market tightness $\theta_{s,t}$. Higher spending per vacancy raises the surplus from an additional match and stimulates hiring decisions. The size of this effect is increasing with the elasticity of matching to spending σ_g.

An increase in spending produces a positive output multiplier effect and an increase in employment. The output multiplier is positive and smaller than one. It is 0.31 on impact and 0.43 after one year. The employment multiplier is positive too and increases after four semesters. The multiplier is larger after 4 semesters than on impact since matching takes time. Positive effects on employment and output are related to the improved efficiency of the labour markets. Spending increases the number of matches and the overall level of employment. In fact, both labour market tightness and the job finding probability increases. There is as well an increase in wages. The Ricardian effect through which higher government spending crowds out private consumption is still at work as consumption drops on impact at -0.11. The search and matching framework also implies that higher interest rate reduces the expected value from an additional match reducing labour demand by firms.

This experiment however shows that the positive spill-over of spending on matching over balances the crowding out of private consumption through an increased efficiency of the labour market. This result is similar with the baseline result of Monacelli *et al.* (2010) except that the rise in employment is not linked to the specificities of the wage bargaining. In Monacelli *et al.* (2010), the positive multiplier effect is related to the crowding out of consumption, which reduces the dis-utility of work activities and lead workers to accept a lower wage.

In line with empirical evidence, wages, labour market tightness and the probability of finding a job increases by 0.2 percent, 1 percent and 2 percent respectively. This results from the positive effect on employment of higher spending and the reduction in searching workers. Contrastingly, the number of vacancies increases on impact before to decline. This result is related to the increases efficiency of matching, whose side effect is to reduce vacancy posting. We attempt further below to reduce this side effect by using CES matching functions.

Thish result must be evaluated against the benchmark case of general government spending, which can be simulated assuming $\sigma_g = 0$ (broken line in Figure 1). An increase in government consumption produces the usual crowding out of private consumption and output in line with the properties of a Ricardian economy. The increase in the interest rate reduces investment and the expected surplus from an additional match leading to a decline in the quantity of goods supplied. The output multiplier is negative at -0.08 after one year.

A matching efficiency shock is modelled by replacing the auto-regressive equation on spending by a similar equation on the matching efficiency σ_m. A matching shock also generates an increase in output and employment, as well as an increase in labour market tightness and the probability of finding a job (dotted line in Figure 1). The main difference with the alternative case is the absence of the resource constraint effect. It follows that both consumption and investment increase following the shock. These two alternative experiments show that the mechanism modelled in this paper is a combination between a crowding-out effect and a matching efficiency effect.

The positive effect of output on the fiscal multiplier holds true for different specifications of the matching function. In particular, two cases are taken into consideration. In the first case, spending are associated with unemployment: $m_t = \sigma_m \left(g_t^{\sigma_g} s_t\right)^{\sigma_s} v_t^{\sigma_v}$ with $\sigma_v = 1 - \sigma_s$. In the second case, spending are associated with vacancies in the matching function: $m_t = \sigma_m s_t^{\sigma_s} \left(g_t^{\sigma_g} v_t\right)^{\sigma_v}$ with $\sigma_v = 1 - \sigma_s$. These two sub-cases capture the idea that spending may target specifically either searching workers or firms. Using the same set of parameters, the output and employment multipliers are both positive (see Figure 2). The multiplier is larger in the case of spending nested with unemployed workers given that σ_s is equal to 0.6. Multipliers are however both smaller than in the case of constant return to scale as the elasticities of matching to spending are smaller. The detail of the computation can be found in Appendix D.

4.2 Crowding-in of consumption:

In Figure 1, consumption declines on impact and then converges monotonically towards its steady state. This feature is at odd with empirical evidence, which points to a positive reaction of consumption to an increase in public spending, although a few exceptions exist as Hall (2009) for instance. Existing models produce an increase in consumption by assuming that the utility function displays complementarity between consumption and a variable adjusting positively to spending. Monacelli and Perotti (2008) and Monacelli *et al.* (2010) assumes complementarity with hours and employment respectively, while Christiano *et al.* (2009) and Ganelli (2003) assume complementarity with government spending. An alternative are models relying on a wealth effect as in perpetual youth model such as Bénassy (2007).

Supply side models imply that consumption is a residual determined by the resource constraint. From eq 34, we know that consumption is given by: $c_t = y_t - x_t - \kappa v_t - g_t$. In a Ricardian model, spending crowds out consumption. However, to the extend that spending have a positive supply side effect, a sufficiently large increase in output may crowd-in consumption. This section explores for which values of the elasticity of matching to spending σ_g and the steady state spending per vacancy

θ_g, crowding-in of consumption takes place.

Figure 3 displays the output and employment multipliers as well as the standard deviation of consumption as a percentage of GDP for different values of σ_g in Panel A and different values of θ_g in Panel B. Figure 3 also reproduces the multipliers on impact (solid line) and after 1 year for output, 3 years for employment and 2 years for consumption (dashed line). Following Baxter and King (1993) and Leeper *et al.* (2010), σ_g fluctuates between 0 and 0.1. θ_g is comprised between 0.1 and 1 corresponding to a spending to GDP ratio between 0.27% and 2.7%, a range which encompasses most OECD countries. In panel A, the output multiplier is zero on impact and negative after 1 year for small values of σ_g since $\sigma_g = 0$ corresponds to the Ricardian case. The size of the multiplier increases with σ_g to the point where consumption is crowded-in on impact for σ_g close to 0.1. In Panel B, the size of the multiplier is decreasing with θ_g due to the diminishing marginal return on spending. For values of θ_g close to 0.1, the output multiplier is close to 1.5 on impact and larger than 2 after 1 year. It follows that consumption is crowded-in for low value of spending to GDP.

Figure 4 reproduces the impulse responses associated with a crowding-in of consumption. The parameters are identical to those of Fig 1 except that σ_g is set to 0.1 in one case (dashed line) and θ_g is set to 0.25 in the other case (solid line). Fiscal multipliers are now 0.56 on impact and 0.83 after one year for $\theta_g = 0.25$ as well as 0.6 on impact and 0.95 after one year for $\sigma_g = 0.1$. It follows that consumption is now responding positively to public spending. The positive reaction of consumption is immediate in the case of higher elasticity of matching to spending, while consumption turns positive after three semesters only in the case of a lower spending to GDP ratio. It seems that the increase in output generated by higher employment crowds in private consumption. The positive impact of public spending on employment boosts output through a supply side effect. The positive impact on consumption that follows over balances the negative impact of higher public spending on private consumption.

4.3 Nominal price rigidities and large multipliers

In this section, the positive effect of spending on matching is studied in interaction with nominal price rigidities. Nominal price rigidities tends to magnify the effect of spending on output and employment through different channels. In the presence of price rigidities, all firms cannot set prices at their optimal level to meet excess demand. Some firms must therefore increase output and labour demand. Ravn *et al.* (2006) for instance use price stickiness together with deep habit formation to produce a positive fiscal multiplier. In a model with search and matching, price rigidities also affects the hiring decisions of firms. The markup $(e_t)^{-1}$ now enters the marginal productivity of labour in the equation of the surplus from an additional match (equation 36). The fall in the markup raises the marginal productivity of labour and leads to higher vacancy posting. Additionally, the discounted value of an additional match depends on the interest rate, which is set by monetary authorities.

$$\frac{\kappa}{q_t} = (1-\eta)(e_t a_t - w^u) + \beta\rho E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{q_{t+1}} (1-\eta p_{t+1}) \right\} \tag{36}$$

Price rigidities are modeled following Trigari *et al.* (2008) and Sala *et al.* (2008). The maximization program of firms described in the previous part of the model now corresponds to the intermediate goods producers. Intermediate goods producers face search costs in the labour market but sell their goods in a competitive market. Intermediate goods are then combined to produce a final good, which is sold in a non-competitive market. The model is also extended to include capacity utilization u_t^c. Capacity utilization requires to distinguish physical capital $k_{p,t}$ and effective capital k_t as well as to include capacity utilization adjustment costs $\Im(u_t^c)$. A difference with the literature cited above is that vacancy costs are kept linear. Introducing prices rigidities requires to modify the equilibrium condition presented in section 2. The new set of equations is presented in the appendix E.[3]

In this section, parameters are identical to Table 1, in particular regarding spending per vacancy $\theta_g = 0.45$ and the elasticity of matching to spending $\sigma_g = 0.1$. The capital adjustment cost η_k is set to 17 and capacity adjustment cost ϕ_u is set to 0.1. The probability that firms cannot adjust their prices to the perfect competition prices is set at $\omega = 0.95$. The price elasticity in the demand function for

[3] The budget constraint of households now looks as follow:

$$c_t + x_t + b_t \leq w_t n_t + w^u(1-n_t) + r_{k,t} u_t^c k_{p,t-1} + r_{t-1} b_{t-1} - \tau_t + \Pi_t - \Im(u_t^c) k_{p,t-1}$$

with $k_t = u_t^c k_{p,t-1}$. Effective capital k_t is a function of physical capital $k_{p,t-1}$ and capacity utilization u_t^c. Capacity utilization are costly to adjust: $\Im(u_t^c) = \frac{r_k}{\phi_u}\left(e^{\phi_u(u_t^c-1)} - 1\right)$. Physical capital accumulates following: $k_{p,t} = (1-\delta)k_{p,t-1} + x_t(1-\phi_t)$. Regarding the equilibrium condition, the main modification is an additional equation for capacity utilization: $r_{k,t} = \frac{\partial \Im}{\partial u_t^c}$. The ressource constraint must also be adjusted to include the costs of capacity utilization $y_t = c_t + x_t + \kappa v_t + \Im(u_t^c) k_{p,t-1}$.

retail goods is conventional and equal to $\varepsilon = 10.091$. The model is closed with a rule for monetary policy (equation 37). The Central Bank adjusts the interest rate to the forward looking inflation at a speed $\phi_\pi = 1.7$, while the sensitivity of the interest rate to the output gap is $\phi_y = 0.7$. Interest rate inertia ρ_m is equal to 0.7.

$$\frac{r_t^n}{r^n} = \left(\frac{r_{t-1}^n}{r^n}\right)^{\rho_m} \left(\frac{E_t\left\{\pi_{t+1}\right\}}{\pi}\right)^{\phi_\pi(1-\rho_m)} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y(1-\rho_m)} \tag{37}$$

Figure 5 displays the dynamic response of output and employment to a spending shock in the presence of price rigidities. The solid line represents the simulations generated by the model for the set of parameters described above. The output multiplier is equal to 1.8 far exceeding unity, while the employment multiplier is 0.5. To distinguish the two channels through which spending affects the output multiplier, the model with sticky price is simulated for $\sigma_g = 0$ (dashed line). In the absence of a matching effect of spending, the output multiplier is lower at 1.2 but still above unity. Although aggregate demand effects appear to have a large impact on the output multiplier, the contribution of the matching channel is still significant. Lastly, the reaction of the interest rate determined by the Taylor rule matters for the output multiplier since it affects the size of the crowding out of private consumption, as well as the discounted value of the surplus from an additional match. Reducing the inertia parameter ρ_m in the Taylor rule from 0.7 to 0.6, significantly reduces the output multiplier from 1.8 to 1.4 (crossed line). In the aftermath of the spending shock, monetary authorities react more quickly to the positive output gap reducing the positive effect of the fiscal shock.

5 Conclusion

This paper has discussed the efficiency of public spending using a DSGE model with search and matching function. The main result is that despite the crowding out of resources, increases in spending yields positive externalities on the labour market and generate positive fiscal multipliers. The multipliers are positive on impact and reach their maximum after 1 year. Spending produces positive supply side effect by easing the matching between unemployed workers and vacancies and increasing labour inputs. The increase in real wage, employment and labour market tightness are also consistent with empirical evidences. A further result is that the size of the multiplier increases with the elasticity of matching to spending and decline with the steady state spending to GDP ratio. For large value of the multiplier, there is a crowding in of private consumption. The model also shows that the output multiplier increases above 1, when nominal price rigidities are added to the model. Interestingly, the matching effect of spending still explains a significant share of the output multiplier in the presence of sticky prices. This theoretical model could be extended with an empirical analysis, which would measure the parameter of the elasticity of matching to spending σ_g as well as the ability of the model to account for historical time series.

References

Baxter, Marianne; King, Robert G. 1993. "Fiscal Policy in General Equilibrium", in *American Economic Review*, Vol. 83, pp. 315–334.

Blanchard, Olivier; Perotti, Roberto. 2002. "An Empirical Characterizion of the Dynamic Effects of Changes in Government Spending and Taxes on Output", in *The Quarterly Journal of Economics*, Vol. 117(4), pp. 1329–1368.

Bénassy, Jean-Pascal. 2007. "Ricardian equivalence and the intertemporal Keynesian multiplier", in *Economics Letters*, Vol. 94(1), pp. 118–123.

Challe, Edouard; Ragot, Xavier. 2011. "Fiscal Policy in a Tractable Liquidity-Constrained Economy", in *The Economic Journal*, Vol. 121(551), pp. 273–317.

Christiano, L. J.; Eichenbaum, M.; Rebelo, S. 2009. "When is the government spending multiplier large?", in *NBER Working Paper*, Vol. 15394, pp. .

Fernández-Villaverde, Jesús. 2010. "Fiscal Policy in a Model With Financial Frictions", in *American Economic Review: Papers & Proceedings*, Vol. 100, pp. 35–40.

Freedman, Charles; Kumhof, Michael; Laxton, Douglas; Muir, Dirk; Mursula, Susanna. 2010. "Global effects of fiscal stimulus during the crisis", in *Journal of Monetary Economics*, Vol. 57, pp. 506–526.

Galí, J.; López-Salido, J. D.; Vallés, J. 2007. "Understanding the effects of government spending on consumption", in *Journal of the European Economic Association*, Vol. 5, No. 1, pp. 227–270.

Ganelli, G. 2003. "Useful government spending, direct crowdingout and fiscal policy interdependence", in *Journal of International Money and Finance*, Vol. 22, pp. 87–103.

Hall, R.E. 2009. "By how much does the GDP rise if the government buys more output?", in *Brookings Papers on Economic Activity*, Vol. 2009 Fall, pp. 183–249.

Leeper, Eric M.; Walker, Todd B.; Yang, Shu-Chun S. 2010. "Government investment and fiscal stimulus", in *Journal of Monetary Economics*, Vol. 57, pp. 1000–1012.

Monacelli, T.; Perotti, R.; Trigari, A. 2010. "Unemployment fiscal multiplier", in *Journal of Monetary Economics*, Vol. ., pp. 1–52. mimeo.

Monacelli, Tommaso; Perotti, Roberto. 2008. "Fiscal Policy, Wealth Effects and Markups", in *NBER Working Paper*, Vol. 14584, pp. 1–56.

Mountford, A.; Uhlig, H. 2009. "What are the effects of fiscal policy shocks?", in *Journal of applied econometrics*, Vol. 24, pp. 960–992.

Ramey, V.; Shapiro, M. 1998. "Costly capital reallocation and the effects of government spending", in *NBER Working Paper*, Vol. 6283.

Ramey, Valerie. 2011. "Identifying government spending shocks: it's all in the timing", in *The Quarterly Journal of Economics*, Vol. 126(1), pp. 1–50.

Ravenna, F.; Walsh, C. E. 2008. "Vacancies, unemployment, and the Phillips curve", in *European Economic Review*, Vol. 52, No. 8, pp. 1494–1521.

Ravn, Morten; Schmitt-Grohé, Stephanie; Uribe, Martin. 2006. "Deep Habits", in *Review of Economic Studies*, Vol. 73, pp. 195–218.

Sala, Luca; Soderstrom, Ulf; Trigari, Antonella. 2008. "Monetary Policy under Uncertainty in an Estimated Model with Labor Market Frictions", in *Journal of Monetary Economics*, Vol. 55 (5),, pp. pp. 983–1006.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", in *American Economic Review*, Vol. 95(1), pp. 25–49.

Trigari, Antonella; Gertler, Mark; Sala, Luca. 2008. "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining", in *Journal of Money, Credit and Banking*, Vol. 40 (8), pp. pp. 1713–1764.

Yuan, Mingwei; Li, Wenli. 2000. "Dynamic employment and hours effects of government spending shocks", in *Journal of Economic Dynamics & Control*, Vol. 24, pp. 1233–1263.

A Detail computation of the model

A.1 Households

From the Bellman equation of households, we get the following Lagrangian:

$$\begin{aligned}\mathcal{L}_t^h &= \frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_t \{H(n_t, k_t, b_t, x_t)\} + \\ &+ \lambda_t \left(w_t n_t + w^u (1-n_t) + r_{k,t} k_{t-1} + r_{t-1} b_{t-1} - \tau_t + \Pi_t - c_t - x_t - b_t\right) + \\ &+ \lambda_t \varphi_t \left((1-\delta) k_{t-1} + x_t (1-\phi_t) - k_t\right) + \mu_t \left(n_t - \rho n_{t-1} - p_t (1-\rho n_{t-1})\right)\end{aligned}$$

The first order condition for consumption links the marginal utility of wealth with the marginal utility of consumption:

$$\lambda_t = \frac{1}{c_t^\sigma}$$

The first order conditions for capital read:

$$\beta E_t \frac{\partial H}{\partial k_t} = \lambda_t \varphi_t$$

From the envelope condition, we know that:

$$E_t \left\{\frac{\partial H}{\partial k_{t-1}}\right\} = \lambda_t \left(r_{k,t} + \varphi_t (1-\delta)\right)$$

We then get the following first order condition for capital:

$$\varphi_t = \beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t} \left[r_{k,t+1} + \varphi_{t+1}(1-\delta)\right]\right\}$$

The first order conditions for investment read:

$$\beta E_t \frac{\partial H}{\partial x_t} - \lambda_t + \lambda_t \varphi_t \left[1 - \frac{\phi}{2}\left(\frac{x_t}{x_{t-1}} - 1\right)^2 - \phi \frac{x_t}{x_{t-1}} \left(\frac{x_t}{x_{t-1}} - 1\right)\right] = 0$$

From the envelope condition, we know that:

$$E_t \left\{\frac{\partial H}{\partial x_{t-1}}\right\} = \lambda_t \varphi_t \phi \left(\frac{x_t}{x_{t-1}} - 1\right) \left(\frac{x_t}{x_{t-1}}\right)^2$$

The equilibrium condition for investment is:

$$\varphi_t\left[1-\left(\frac{\phi}{2}\left(\frac{x_t}{x_{t-1}}-1\right)^2+\frac{x_t}{x_{t-1}}\phi\left(\frac{x_t}{x_{t-1}}-1\right)\right)\right] = 1-\beta E_t\left\{\varphi_{t+1}\Lambda_{t,t+1}\left(\frac{x_{t+1}}{x_t}\right)^2\phi\left(\frac{x_t}{x_{t-1}}-1\right)\right\}$$

with $\Lambda_{t,t+1}$is defined as $\frac{\lambda_{t+1}}{\lambda_t}$. The first order condition with respect to public bonds can be expressed as follow:

$$\beta_t E_t\left\{\frac{\partial H}{\partial b_t}\right\} = \lambda_t$$

From the envelope condition, we know:

$$\frac{\partial H}{\partial b_{t-1}} = \lambda_t r_{t-1}$$

We get the following first order condition for public bonds:

$$\frac{1}{r_t} = \beta_t E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\right\}$$

We now derive $H_{n,t}$ the representative household's marginal value of having one of its member hired in the labor market rather than unemployed. The first derivative of the lagrangian with respect to n_t is:

$$\beta\frac{\partial H}{\partial n_t}+\lambda_t\left(w_t-w^u\right)+\mu_t = 0$$

From the envelope condition, we know:

$$\frac{\partial H}{\partial n_{t-1}} = -\mu_t\rho\left(1-p_t\right)$$

Combining the two conditions and defining $H_{n,t}=-\mu_t$, we get:

$$H_{n,t} = \lambda_t\left(w_t-w^u\right)+\beta\rho E_t\left\{H_{n,t+1}\left(1-p_{t+1}\right)\right\}$$

A.2 Firms

From the Bellman equation of firms, we get the following Lagrangian:

$$\mathscr{L}_t^f = \left(k_{t-1}\right)^{\zeta}\left(n_t\right)^{1-\zeta}-w_t n_t-r_{k,t}k_{t-1}-\kappa v_t+\beta E_t\left\{\Lambda_{t,t+1}F\left(n_t,k_t\right)\right\}+\psi_t\left[n_t-\rho n_{t-1}-q_t v_t\right]$$

The first order conditions of the firm's optimization problem with respect to k_{t-1} gives:

$$r_{k,t} = \zeta \frac{y_t}{k_{t-1}}$$

The first order conditions of the firm's optimization problem with respect to v_t gives:

$$\psi_t = -\frac{\kappa}{q_t}$$

The first order conditions of the firm's optimization problem with respect to n_t gives:

$$a_t - w_t + \beta E_t \left\{ \Lambda_{t,t+1} \frac{\partial F}{\partial n_t} \right\} + \psi_t = 0$$

From the envelope condition, we know:

$$\frac{\partial F}{\partial n_{t-1}} = -\psi_t \rho$$

Combining the two conditions and defining $F_{n,t} = -\psi_t$ gives us the value for the firms of an additional match:

$$F_{n,t} = a_t - w_t + \beta \rho E_t \left\{ \Lambda_{t,t+1} F_{n,t+1} \right\}$$

From above, we know that $\psi_t = -\frac{\kappa}{q_t}$, which gives us the equilibrium condition for employment.

$$\frac{\kappa}{q_t} = a_t - w_t + \beta \rho E_t \left\{ \Lambda_{t,t+1} \frac{\kappa}{q_{t+1}} \right\}$$

A.3 Wage bargaining

Wages are determined through a Nash equilibrium:

$$w_t = max \left\{ (H_{n,t})^{\eta} (F_{n,t})^{1-\eta} \right\}$$

The first order condition is:

$$\begin{aligned} \eta (H_{n,t})^{\eta-1} \frac{\partial H_{n,t}}{\partial w_t} (F_{n,t})^{1-\eta} + H_{n,t}^{\eta} (1-\eta) (F_{n,t})^{-\eta} \frac{\partial F_{n,t}}{\partial w_t} &= 0 \\ \eta (H_{n,t})^{-1} \lambda_t (F_{n,t})^{\eta} &= (1-\eta) \\ F_{n,t} &= \frac{(1-\eta)}{\eta} \frac{H_{n,t}}{\lambda_t} \end{aligned}$$

The reservation wage of firms $\bar{w}_t$ and workers $\underline{w}_t$ are simply found by setting $F_{n,t}=0$ and $H_{n,t}=0$ and solving for w_t.

$$\bar{w}_t = a_t + \beta\rho E_t\left\{\Lambda_{t,t+1}F_{n,t+1}\right\}$$

$$\underline{w}_t = w^u - \beta\rho E_t\left\{\Lambda_{t,t+1}\frac{H_{n,t+1}}{\lambda_{t+1}}(1-p_{t+1})\right\}$$

Wage is the weighted sum of both reservation wages:

$$\begin{aligned}
w_t &= \eta\bar{w}_t + (1-\eta)\underline{w}_t \\
&= \eta a_t + (1-\eta)w^u + \eta\beta\rho E_t\left\{\Lambda_{t,t+1}F_{n,t+1}\right\} - (1-\eta)\beta\rho E_t\left\{\Lambda_{t,t+1}\frac{H_{n,t+1}}{\lambda_{t+1}}(1-p_{t+1})\right\} \\
&= \eta a_t + (1-\eta)w^u + \eta\beta\rho E_t\left\{\Lambda_{t,t+1}F_{n,t+1}p_{t+1}\right\} \\
&= \eta a_t + (1-\eta)w^u + \eta\beta\rho\kappa E_t\left\{\Lambda_{t,t+1}\frac{q_{t+1}}{p_{t+1}}\right\}
\end{aligned}$$

using $(1-\eta)\frac{(H_{n,t+1})}{\lambda_{t+1}} = \eta\left(F_{n,t+1}\right) = \eta\frac{\kappa}{q_{t+1}}$.
The total surplus $S_{n,t}$ is given by:

$$\begin{aligned}
S_{n,t} &= (a_t - w^u) + \beta\rho E_t\left\{\Lambda_{t,t+1}F_{n,t+1}\right\} + \beta\rho E_t\left\{\Lambda_{t,t+1}\frac{H_{n,t+1}}{\lambda_{t+1}}(1-p_{t+1})\right\} \\
S_{n,t} &= (a_t - w^u) + \beta\rho E_t\left\{\Lambda_{t,t+1}\left[F_{n,t+1} + \frac{H_{n,t+1}}{\lambda_{t+1}}(1-p_{t+1})\right]\right\} \\
S_{n,t} &= (a_t - w^u) + \beta\rho E_t\left\{\Lambda_{t,t+1}\left[F_{n,t+1} + \frac{\eta}{(1-\eta)}F_{n,t+1}(1-p_{t+1})\right]\right\} \\
S_{n,t} &= (a_t - w^u) + \beta\rho E_t\left\{\Lambda_{t,t+1}\frac{F_{n,t+1}}{(1-\eta)}(1-\eta p_{t+1})\right\} \\
S_{n,t} &= (a_t - w^u) + \beta\rho E_t\left\{\Lambda_{t,t+1}S_{n,t+1}(1-\eta p_{t+1})\right\}
\end{aligned}$$

Plugging in $S_{n,t} = \frac{F_{n,t}}{(1-\eta)} = \frac{\kappa}{(1-\eta)q_{t+1}}$, we get the equilibrium condition in the labour market.

$$\frac{\kappa}{q_t} = (1-\eta)(a_t - w^u) + \beta\rho E_t\left\{\Lambda_{t,t+1}\frac{\kappa}{q_{t+1}}(1-\eta p_{t+1})\right\}$$

The ressource constraint is found by summing the budget constraints of households, firms and government:

$$\begin{aligned}
&c_t + x_t + b_t - w_t n_t - w^u(1-n_t) - r_{k,t}k_{t-1} - r_{t-1}b_{t-1} + \\
&\qquad \tau_t - \Pi_t + \Pi_t - y_t + w_t n_t + r_{k,t}k_{t-1} + \kappa v_t = b_t - r_{t-1}b_{t-1} - d_t - w^u(1-n_t) + \tau_t
\end{aligned}$$

$$
\begin{aligned}
c_t + x_t - y_t + \kappa v_t &= -g_t \\
y_t &= c_t + x_t + \kappa v_t + g_t
\end{aligned}
$$

B Equilibrium conditions

There are 20 equations in the model:

$$
\begin{aligned}
q_t &= \sigma_m \theta_{s,t}^{-\sigma_s} \theta_{g,t}^{\sigma_g} \\
p_t &= \sigma_m \theta_{s,t}^{1-\sigma_s} \theta_{g,t}^{\sigma_g} \\
\theta_{s,t} &= \frac{v_t}{1-\rho n_{t-1}} \\
\theta_{g,t} &= \frac{g_t}{v_t} \\
k_t &= (1-\delta) k_{t-1} + x_t \left(1 - \frac{\eta_k}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2 \right) \\
y_t &= k_{t-1}^{\zeta} n_t^{1-\zeta} \\
n_t &= \rho n_{t-1} + q_t v_t \\
\lambda_t &= \frac{1}{c_t^{\sigma}} \\
\varphi_t &= \left[1 - \beta E_t \left(\varphi_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{x_{t+1}}{x_t} \right)^2 \eta_k \left(\frac{x_{t+1}}{x_t} - 1 \right) \right) \right] / \left[1 - \left(\frac{\eta_k}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2 + \frac{x_t}{x_{t-1}} \eta_k \left(\frac{x_t}{x_{t-1}} - 1 \right) \right) \right] \\
\varphi_t &= \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left[r_{k,t+1} + \varphi_{t+1} (1-\delta) \right] \right) \\
r_{k,t} &= \zeta \frac{y_t}{k_{t-1}} \\
a_t &= (1-\zeta) \frac{y_t}{n_t} \\
y_t &= c_t + x_t + \kappa v_t + g_t \\
w_t &= \eta a_t + (1-\eta) w^u + \eta \beta \kappa E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \theta_{s,t+1} \right\} \\
\frac{\kappa}{q_t} &= (1-\eta)(a_t - w^u) + \beta \rho E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{q_{t+1}} (1 - \eta p_{t+1}) \right\} \\
b_t &= r_{t-1} b_{t-1} + d_t \\
d_t &= g_t + w^u (1 - n_t) - \tau_t \\
g_t &= (1-\rho_g) g + \rho_g g_{t-1} + \varepsilon_{I,t} \\
\tau_t &= (1-\rho_\tau) \tau + \rho_\tau \tau_{t-1} + \tau_b (b_{t-1} - b) \\
\frac{1}{r_t} &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\}
\end{aligned}
$$

C Steady state

Table 2: Steady state - part 1

$\varphi = 1$	$r_k = 1/\beta - 1 - \delta$	$r = 1/\beta$
$\theta_s = 0.5$	$\theta_g = 0.45$	$p = 0.45$
$\sigma_m = \frac{p}{\theta_s^{1-\sigma_s}\theta_g^{\sigma_g}}$	$q = \sigma_m \theta_s^{-\sigma_s}\theta_g^{\sigma_g}$	$v = \frac{\theta_s(1-\rho)}{1-\rho+\rho q \theta_s}$
$n = \frac{qv}{1-\rho}$	$\frac{y}{k} = \frac{r_k}{\zeta}$	$k = n\left(\frac{y}{k}\right)^{\frac{1}{\zeta-1}}$
$y = \left(\frac{y}{k}\right)k$	$x = \delta k$	$a = (1-\zeta)\frac{y}{n}$

At this stage, we get the steady state values of $w^u$and κ by solving the equations 27 and 29. We then have:

Table 3: Steady state - part 2

$w = w^u/\alpha_u$	$g = \frac{v}{\theta_g}$	$c = y - x - \kappa v - g$
$\lambda = \frac{1}{c}$	$b = 0.6y$	
$\tau = g + w^u(1-n) - (1-r)b$	$d = b/(1-r)$	

D Nested matching function

This appendix discusses the impact on the fiscal multiplier of different types of matching function. The baseline matching function has constant return to scale in s_t, v_t and g_t with $\sigma_s + \sigma_v + \sigma_g = 1$. We here briefly discuss the impact of two alternative specification. In a first case, spending are nested with searching workers and captures the idea that active labour market spending may be directed towards unemployed:

$$m_t = \sigma_m \left(g_t^{\sigma_g} s_t\right)^{\sigma_s} v_t^{1-\sigma_s}$$

with $\sigma_v = 1 - \sigma_s$ The matching function has constant return on unemployment and vacancies and decreasing return on active labour market policies. For convenience, we use the ratio $\theta_{s,t} = \frac{v_t}{s_t}$ to measure labour market tightness:

$$\begin{aligned} q_t &= \sigma_m \theta_{s,t}^{-\sigma_s} g_t^{\sigma_g \sigma_s} \\ p_t &= \sigma_m \theta_{s,t}^{1-\sigma_s} g_t^{\sigma_g \sigma_s} \end{aligned}$$

In a second case, spending are nested with vacancies to capture the idea that labour market policies also target firms:

$$m_t = \sigma_m s_t^{\sigma_s} \left(g_t^{\sigma_g} v_t \right)^{1-\sigma_s}$$

with $\sigma_v = 1 - \sigma_s$. The matching function has constant return on unemployment and vacancies and decreasing return on active labour market policies. For convenience, we use the ratio $\theta_{s,t} = \frac{v_t}{s_t}$ to measure labour market tightness:

$$\begin{aligned} q_t &= \sigma_m \theta_{s,t}^{-\sigma_s} g_t^{\sigma_g(1-\sigma_s)} \\ p_t &= = \sigma_m \theta_{s,t}^{1-\sigma_s} g_t^{\sigma_g(1-\sigma_s)} \end{aligned}$$

E Equilibrium conditions with nominal price rigidities

The equilibrium condition of the model with sticky prices are as follow:

$$\begin{aligned}
q_t &= \sigma_m \theta_{s,t}^{-\sigma_s} \theta_{g,t}^{\sigma_g} \\
p_t &= \sigma_m \theta_{s,t}^{1-\sigma_s} \theta_{g,t}^{\sigma_g} \\
\theta_{s,t} &= \frac{v_t}{1-\rho n_{t-1}} \\
\theta_{g,t} &= \frac{g_t}{v_t} \\
k_{p,t} &= (1-\delta) k_{p,t-1} + x_t \left(1 - \frac{\eta_k}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2 \right) \\
y_t^w &= k_t^{\zeta} n_t^{1-\zeta} \\
n_t &= \rho n_{t-1} + q_t v_t \\
\lambda_t &= \frac{1}{c_t^{\sigma}} \\
\varphi_t &= \left[1 - \beta E_t \left(\varphi_{t+1} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{x_{t+1}}{x_t} \right)^2 \eta_k \left(\frac{x_{t+1}}{x_t} - 1 \right) \right) \right] / \left[1 - \left(\frac{\eta_k}{2} \left(\frac{x_t}{x_{t-1}} - 1 \right)^2 + \frac{x_t}{x_{t-1}} \eta_k \left(\frac{x_t}{x_{t-1}} - 1 \right) \right) \right] \\
\varphi_t &= \beta E_t \left(\frac{\lambda_{t+1}}{\lambda_t} \left[r_{k,t+1} u_{t+1}^c - \Im(u_{t+1}^c) + \varphi_{t+1} (1-\delta) \right] \right) \\
r_{k,t} &= e_t \zeta \frac{y_t^w}{k_t} \\
a_t &= (1-\zeta) \frac{y_t^w}{n_t} \\
y_t &= c_t + x_t + \kappa v_t + g_t + \Im(u_t^c) k_{p,t-1} \\
w_t &= \eta e_t a_t + (1-\eta) w^u + \eta \beta \kappa E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \theta_{s,t+1} \right\} \\
\frac{\kappa}{q_t} &= (1-\eta)(e_t a_t - w^u) + \beta \rho E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{q_{t+1}} (1 - \eta p_{t+1}) \right\}
\end{aligned}$$

$$
\begin{aligned}
b_t &= r_{t-1} b_{t-1} + d_t \\
d_t &= g_t + w^u (1 - n_t) - \tau_t \\
g_t &= (1 - \rho_g) g + \rho_g g_{t-1} + \varepsilon_{I,t} \\
\tau_t &= (1 - \rho_\tau) \tau + \rho_\tau \tau_{t-1} + \tau_b (b_{t-1} - b) \\
1 &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{r_t^n}{\pi_{t+1}} \right\} \\
k_t &= u_t^c k_{p,t-1} \\
r_{k,t} &= \frac{\partial \Im}{\partial u_t^c} \\
f_t^1 &= \frac{\varepsilon - 1}{\varepsilon} f_t^2 \\
f_t^1 &= \tilde{p}_t^{-1-\varepsilon} y_t e_t + \beta \chi E_t \left(\frac{\lambda_{t+1}^o}{\lambda_t^o} \pi_{t+1}^{\varepsilon} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-1-\varepsilon} f_{t+1}^1 \right) \\
f_t^2 &= \tilde{p}_t^{-\varepsilon} y_t + \beta \chi E_t \left(\frac{\lambda_{t+1}^o}{\lambda_t^o} \pi_{t+1}^{\varepsilon-1} \left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}} \right)^{-\varepsilon} f_{t+1}^2 \right) \\
\frac{r_t^n}{r^n} &= \left(\frac{r_{t-1}^n}{r^n} \right)^{\rho_m} \left(\frac{E_t \{\pi_{t+1}\}}{\pi} \right)^{\phi_\pi (1-\rho_m)} \left(\frac{y_t}{y_{t-1}} \right)^{\phi_y (1-\rho_m)} \\
1 &= \chi \pi_t^{\varepsilon-1} + (1 - \chi) \tilde{p}_t^{1-\varepsilon} \\
y_t^w &= s_t y_t \\
s_t &= (1 - \chi) \tilde{p}_t^{-\varepsilon} + \chi \pi_t^{\varepsilon} s_{t-1}
\end{aligned}
$$

F Figures

Figure 1: Baseline case: labour market spill-over

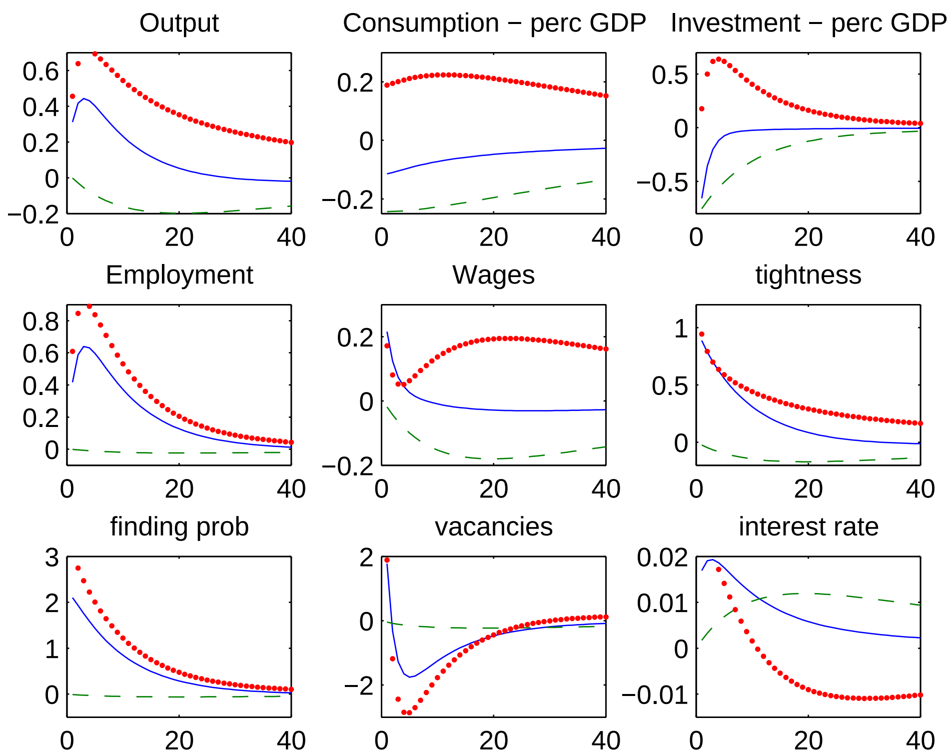


Figure 2: Nested matching function

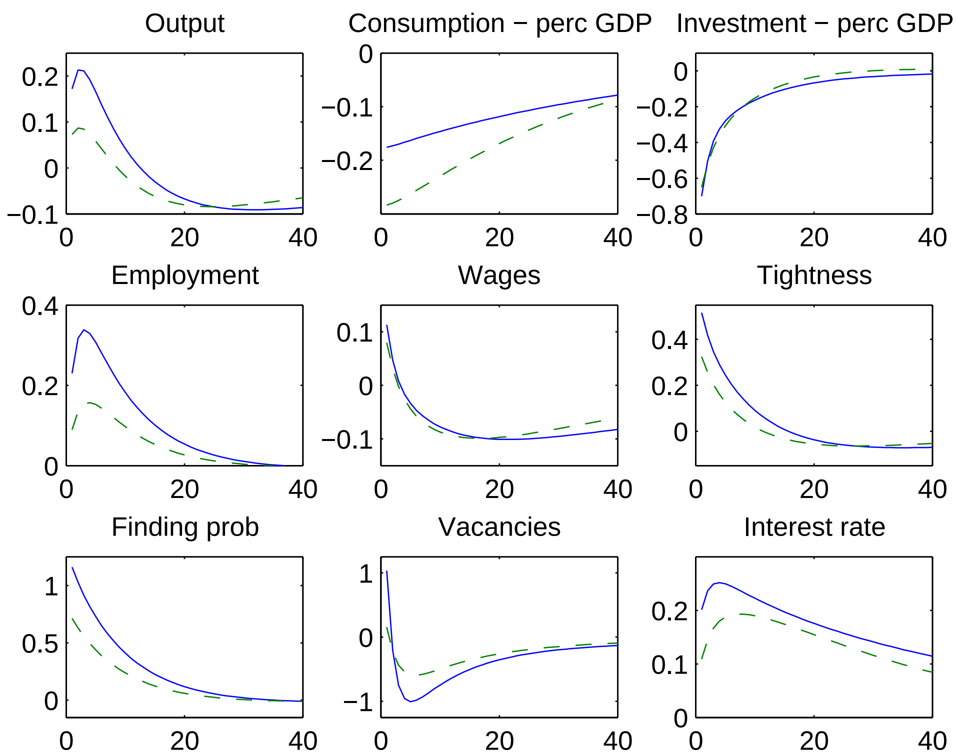
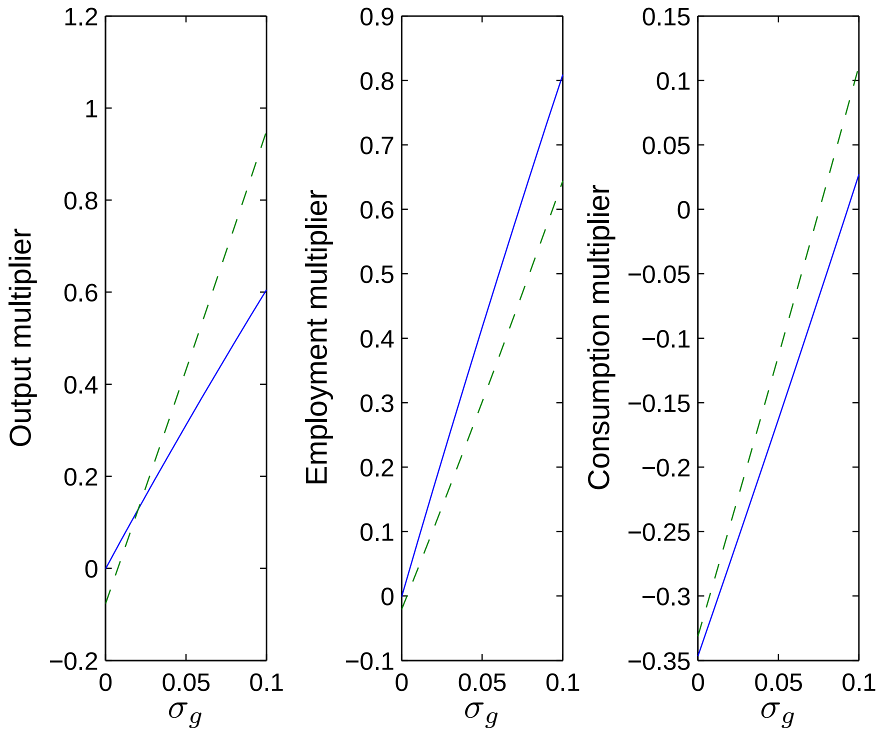
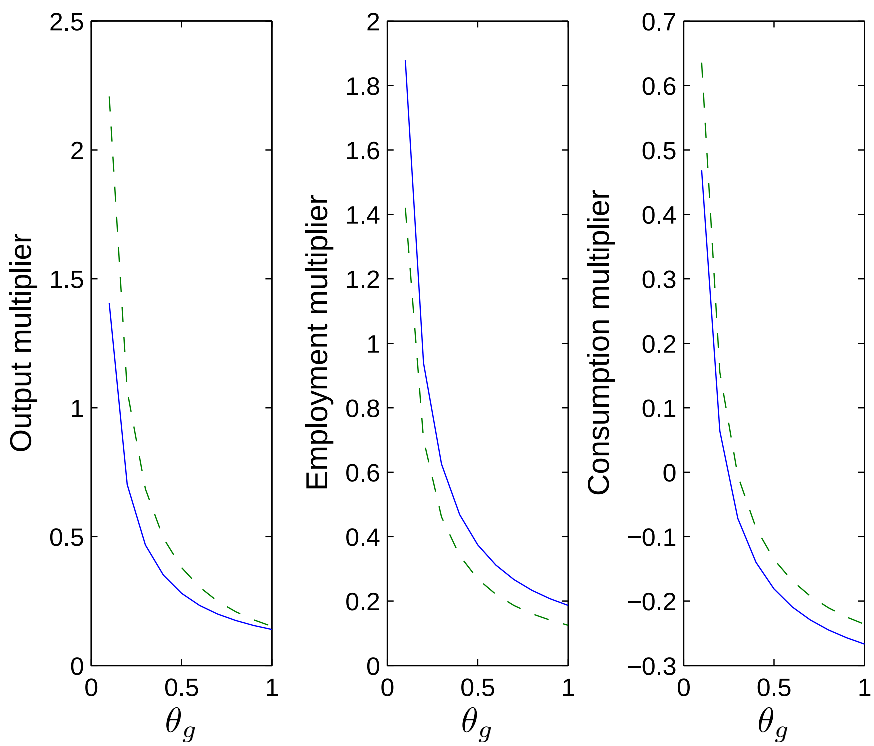


Figure 3: Sensitivity analysis w.r.t. σ_g and θ_g



(a) $0 < \sigma_g < 0.1$



(b) $0.27\% < \frac{g}{y} < 2.7\%$

Figure 4: Crowding-in of private consumption

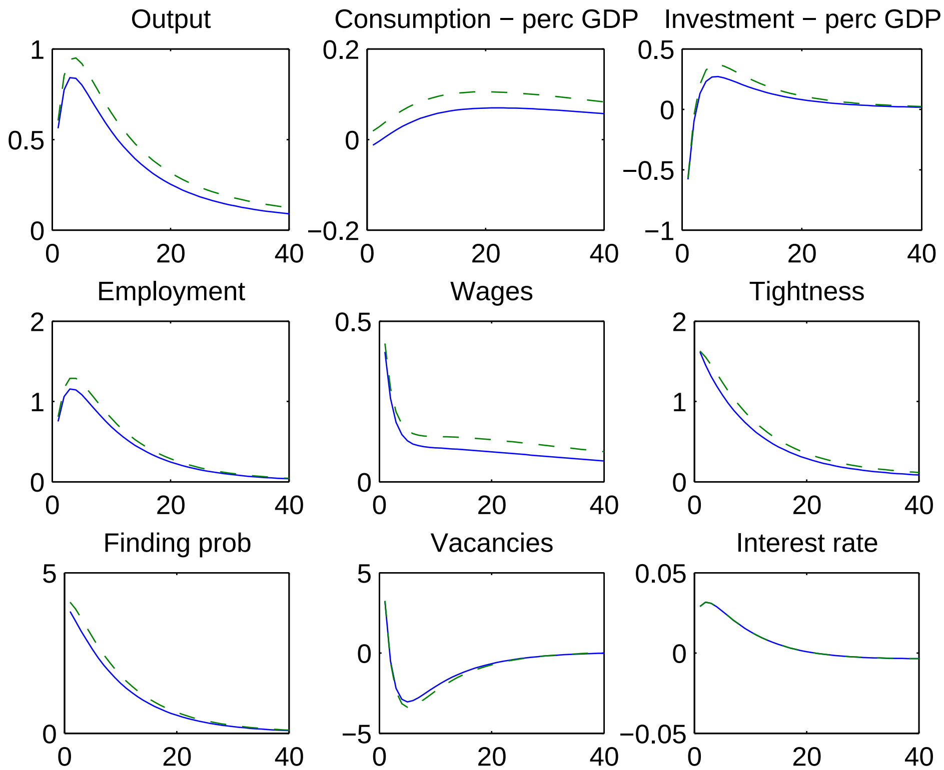


Figure 5: Nominal price rigidities